Computational Aeroacoustics by the Space-Time CE/SE Method

by Ching Y. Loh
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The Space-Time Conservation Element and Solution Element Method, or CE/SE Method for short, is a newly developed numerical method for conservation laws. Despite its second order accuracy, it possesses low dispersion errors and low dissipation. The method is robust enough to cover a wide spectrum of compressible flows: from weak linear acoustic waves to strong discontinuous waves (shocks). An outstanding feature of the CE/SE scheme is its novel, simple but effective non-reflecting boundary condition (NRBC), which is particularly valuable for CAA (computational aeroacoustics).

In this seminar, the 1-D and 2-D unstructured version of the CE/SE schemes are first briefly described. Secondly, some discussions on the NRBC are given. Then, various exmples for linear, nonlinear aeroacoustics are presented. These include (but not limited to):

1. CAA Workshop problems (compared with exact solutions),
2. shear layer instbility, vortex roll-up (compared with linear theory)
3. jet noise - Mach radiation and jet as a Mach wave guide.
4. vortex-shock interaction, vortex-blade interaction (BVI).
5. jet noise - screech of underexpanded jet (compared to experiment),
6. jet noise - 3-D underexpanded rectangular jet,
7. aeroacoustic feedback system - cavity noise, whistling nozzle.

At last, if time available, video animations will be shown for some of the above examples.

Computational Aeroacoustics by the Space-Time CE/SE Method

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Review of the CE/SE Methodology

(S.C. Chang, J.C.P., Vol. 119, 295-324, 1995)

A. 1-D CE/SE Method

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = 0$$

$$\oint_{S(CE)} \mathbf{h}_m^* \cdot d\mathbf{s} = 0, \qquad m = 1, 2, ..., M$$

$$\mathbf{h}_m^* = (f_m^*, u_m^*), \quad m = 1, 2, ..., M$$

CE= conservation element = a control volume,

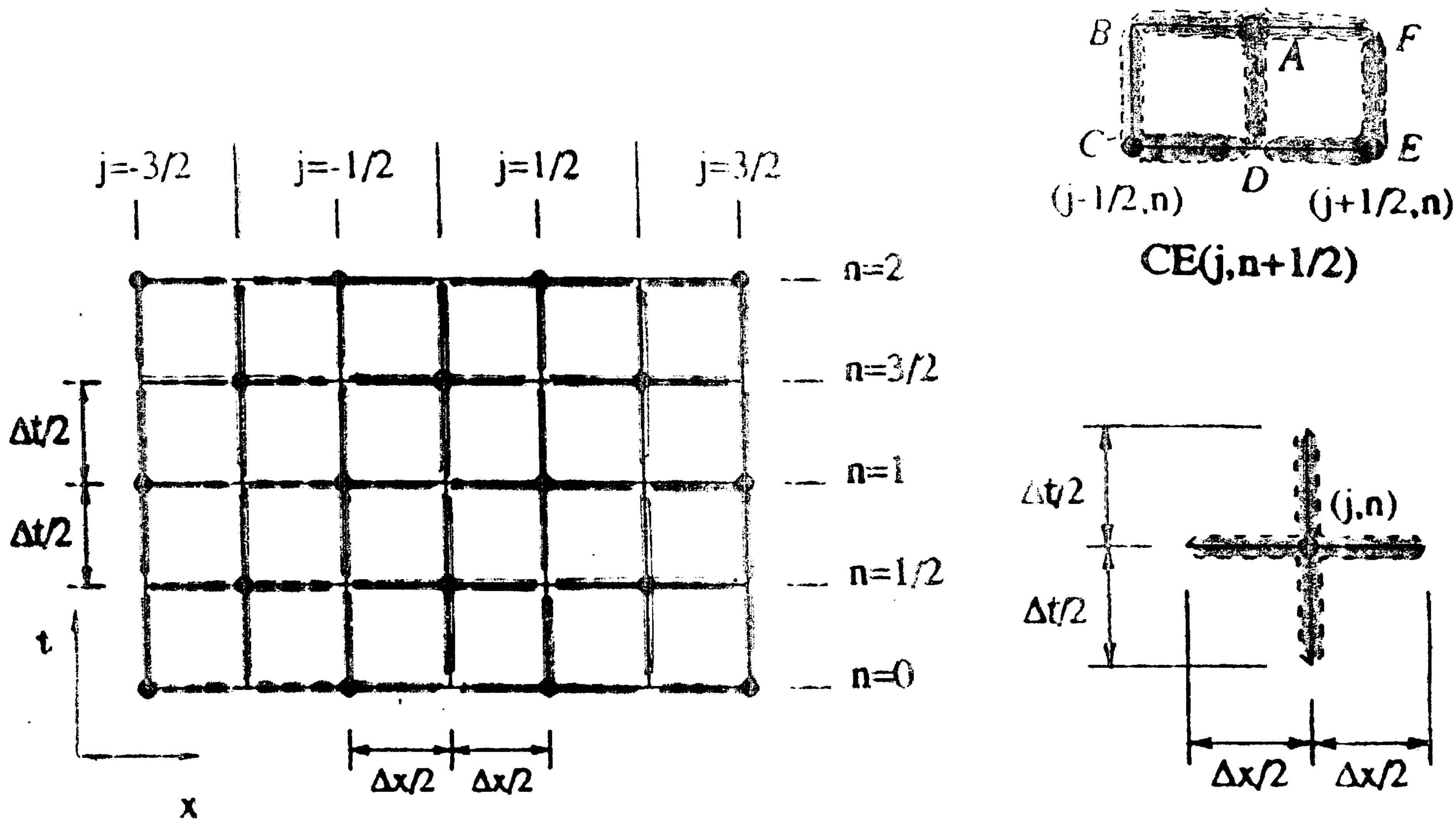
SE = solution element = cell interface. they can be related only by conservation laws.

Unknowns: $\mathbf{U}$, $\mathbf{U}_x$ defined on SEs.

For fluxes on SE, $\mathbf{U}$ and $\mathbf{F}$ are expanded in Taylor series:

$$\mathbf{U}^*(x, t; j, n) = \mathbf{U}_j^n + (\mathbf{U}_x)_j^n (x - x_j) + (\mathbf{U}_t)_j^n (t - t^n)$$

$$\mathbf{F}^*(x, t; j, n) = \mathbf{F}_j^n + (\mathbf{F}_x)_j^n (x - x_j) + (\mathbf{F}_t)_j^n (t - t^n)$$



j=-3/2
j=-1/2
j=1/2
j=3/2
n=2
n=3/2
n=1
n=1/2
n=0
Δt/2
Δt/2
t
x
Δx/2
Δx/2
B
A
F
C
D
E
(j-1/2,n)
(j+1/2,n)
(j,n)
Δt/2
Δt/2
Δx/2
Δx/2

CE(j,n+1/2)

SE(j,n)

Review of the Unstructured CE/SE Euler Solver

(For CE/SE method, see S.C.Chang *et al*, J.C.P. V. ~~119~~ 156, pp ~~295–324~~ 89–136, 1999)

The CE/SE method is naturally adapted to unstructured grid.

The Euler equations in vector form:

$$\mathbf{U}_t + \mathbf{F}_x + \mathbf{G}_y = \mathbf{0}$$

$$\mathbf{U} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix}.$$

Note: $\mathbf{F} = \mathbf{F}(\mathbf{U})$, $\mathbf{G} = \mathbf{G}(\mathbf{U})$

The integral form

$$\oint_{S(V)} \mathbf{h_m} \cdot d\mathbf{s} = 0, \qquad \mathbf{h_m} = \left(f_m^*, g_m^*, u_m^*\right), \qquad m = 1, 2, 3, 4$$

Taylor expansions are used here for flux computations.

$$\mathbf{U}^*(x,y,t) = \mathbf{U}_0^n + (\mathbf{U}_x)_0^n(x - x_0) + (\mathbf{U}_y)_0^n(y - y_0) + (\mathbf{U}_t)_0^n(t - t^n)$$

$$\mathbf{F}^*(x,y,t) = \mathbf{F}_0^n + (\mathbf{F}_x)_0^n(x - x_0) + (\mathbf{F}_y)_0^n(y - y_0) + (\mathbf{F}_t)_j^n(t - t^n)$$

$$\mathbf{G}^*(x,y,t) = \mathbf{G}_0^n + (\mathbf{G}_x)_0^n(x - x_0) + (\mathbf{G}_y)_0^n(y - y_0) + (\mathbf{G}_t)_0^n(t - t^n)$$

CE= conservation element = a control volume,

SE = solution element = cell interface.

Unknowns: $\mathbf{U}$, $\mathbf{U}_x$, $\mathbf{U}_y$ defined on SEs.

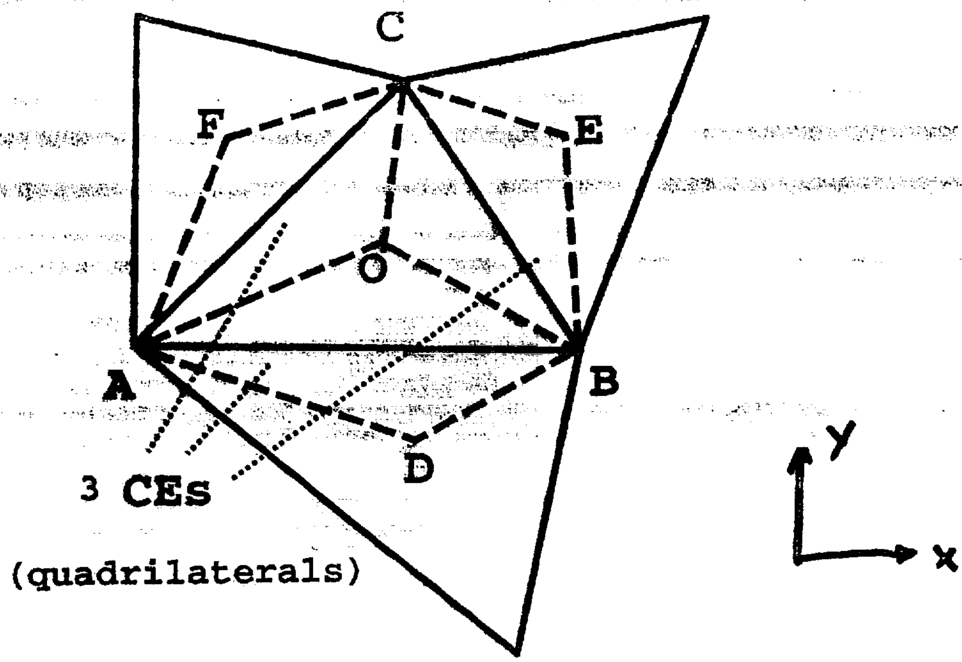
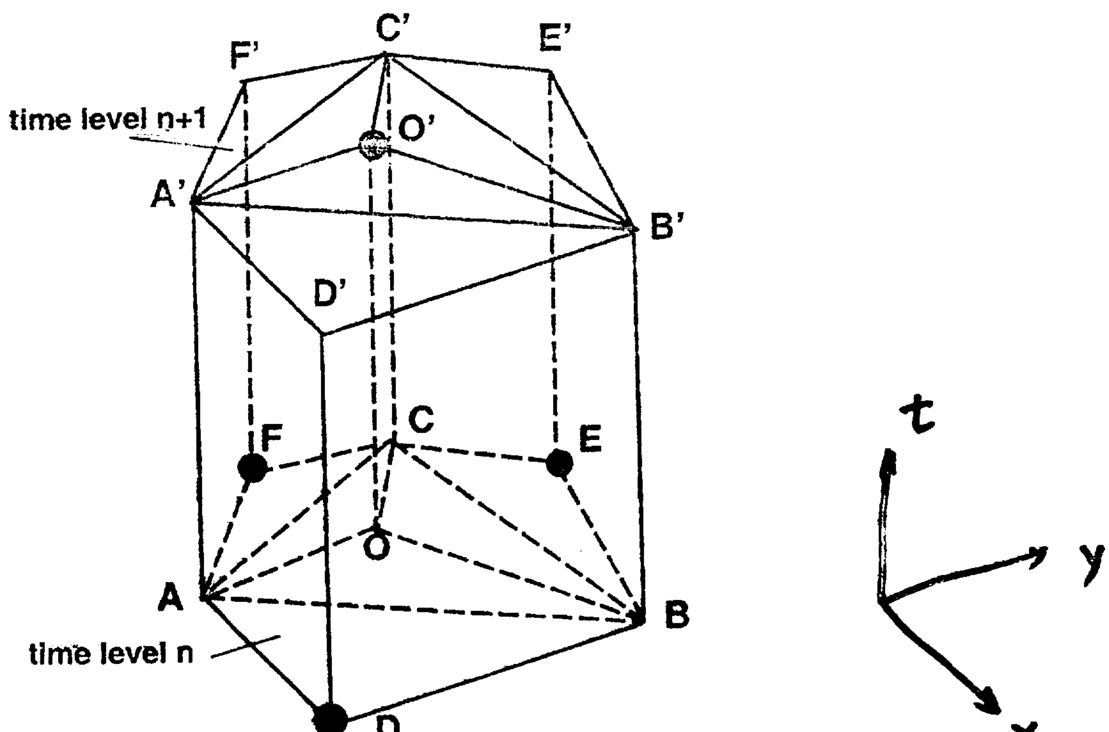


Fig. 1: basic CE/SE grid structure



hexagon cyliner ADBECFA – A'D'B'E'C'F'A' and its 3 CEs, OADB–O'A'D'B', OBEC–O'B'E'C', OCFA – O'C'F'A'

The Non-reflecting boundary Conditions

Type I:

(i) $(u_m)_j^{n+1/2} = (u_m)_{j'}^n$ (simple extrpolation),

(ii) $(u_{mx})_j^{n+1/2} = 0,\ m = 1, 2, 3, 4$

(iii) $(u_{my})_j^{n+1/2} = 0,\ m = 1, 2, 3, 4$

or (at outflow)

Type II:

(i) $(u_m)_j^{n+1/2} = (u_m)_{j'}^n$

(ii) $(u_{mx})_j^{n+1/2} = 0,\ m = 1, 2, 3, 4$

(iii) $(u_{my})_j^{n+1/2} = (u_{my})_{j'}^n,\ m = 1, 2, 3, 4$

The Buffer Zone at jet Outflow Boundary (optional)

For an extremely clean acoustic filed, choose a buffer zone of 5-30 cells with rapid increased size.

Examples with exact solution:

CAA Workshop I:

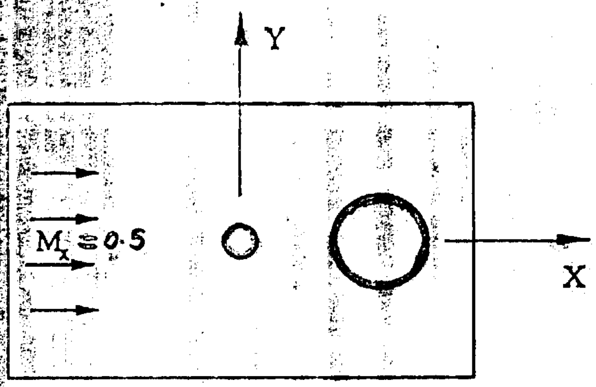


Figure for Problem 1, Category 3

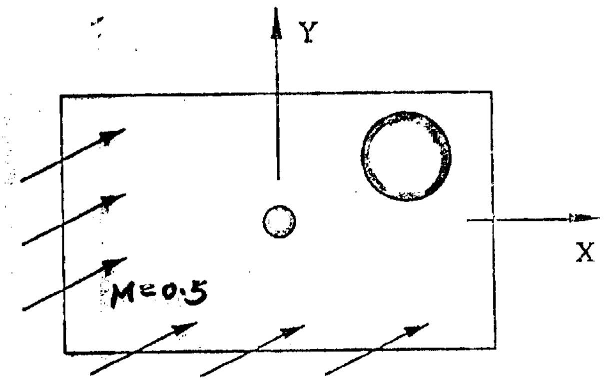


Figure for Problem 2, Category 3

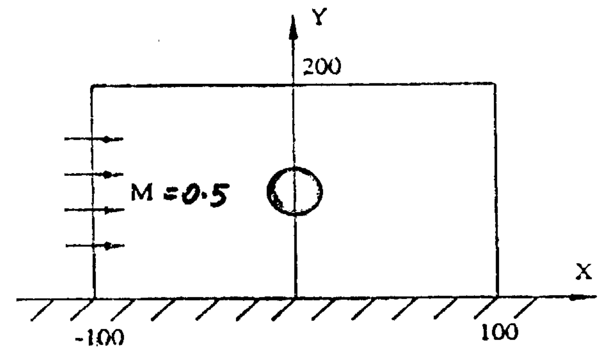
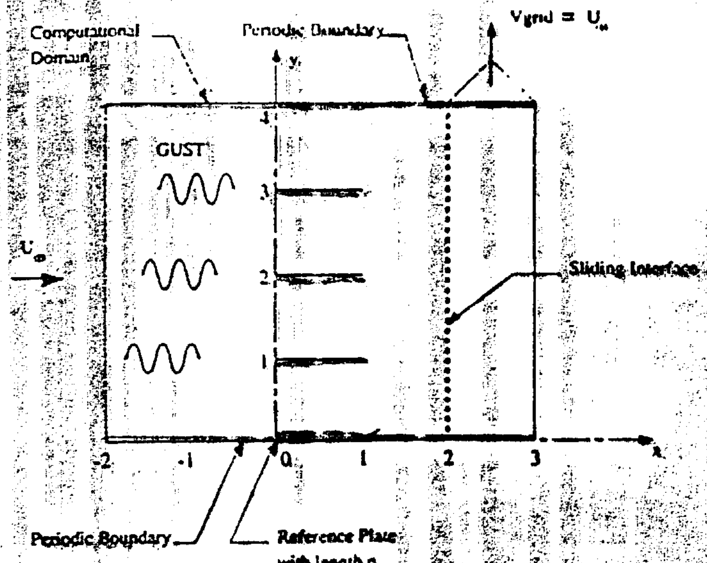
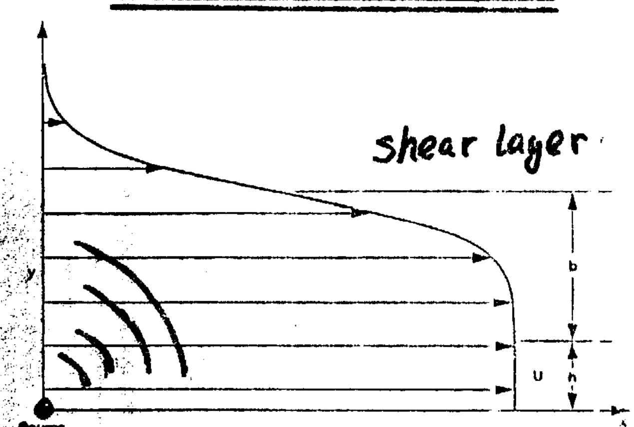


Figure for Problem 1, Category 4

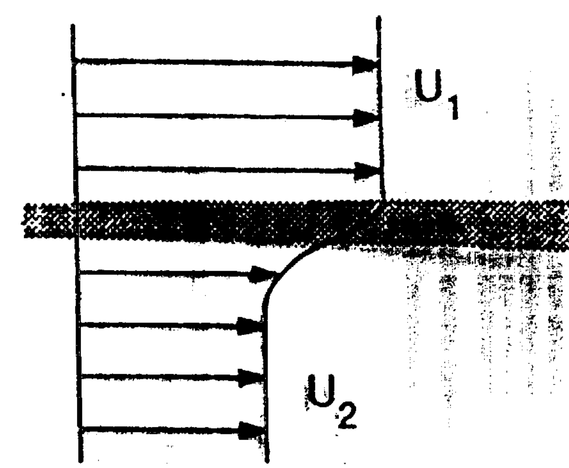
CAA workshop II: Cat. 3

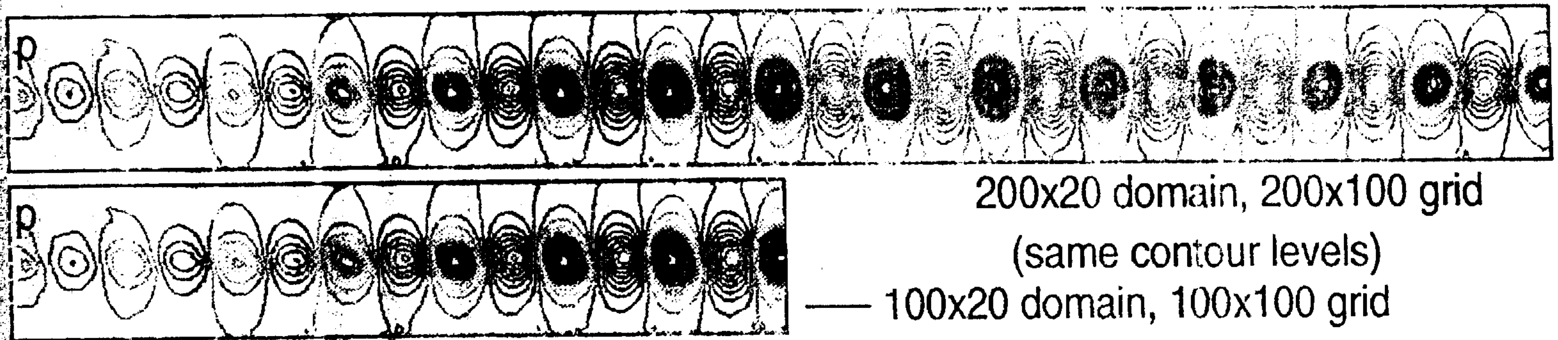


CAA Workshop III: Cat. 5



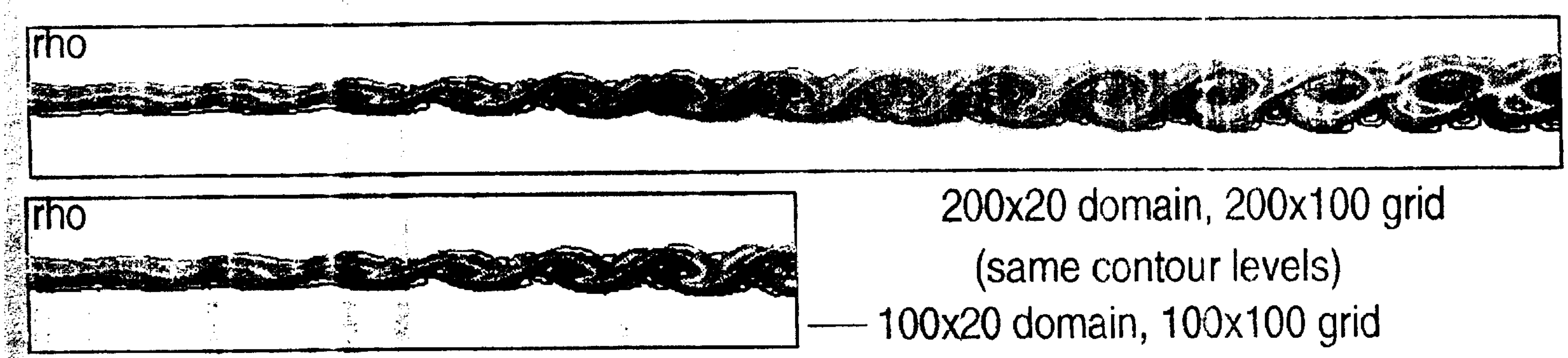
Shear layer instability vortex roll-up





200x20 domain, 200x100 grid
(same contour levels)
— 100x20 domain, 100x100 grid

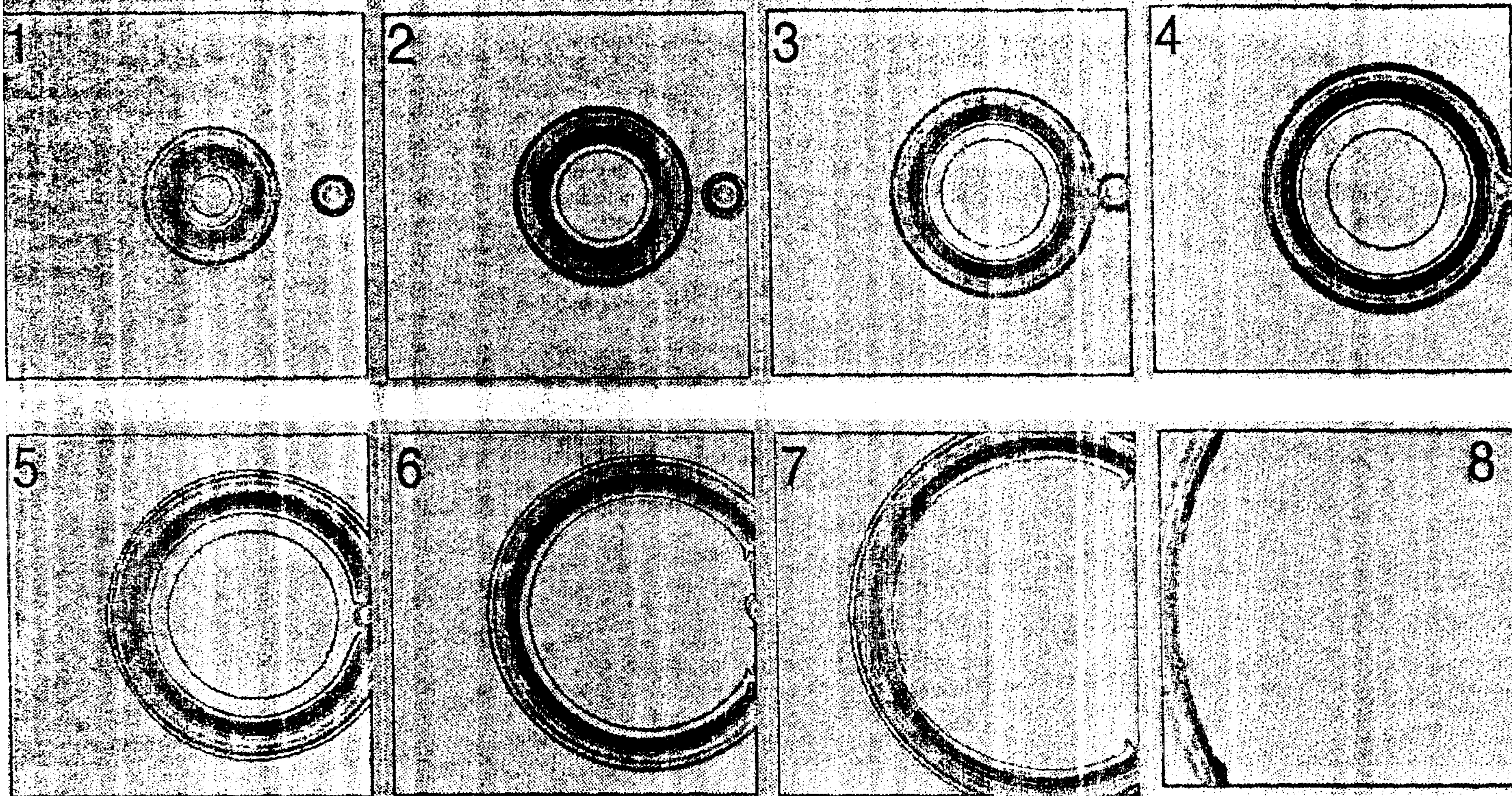
comparison of numerical results with different computational domains, showing effectiveness of outflow NRBC



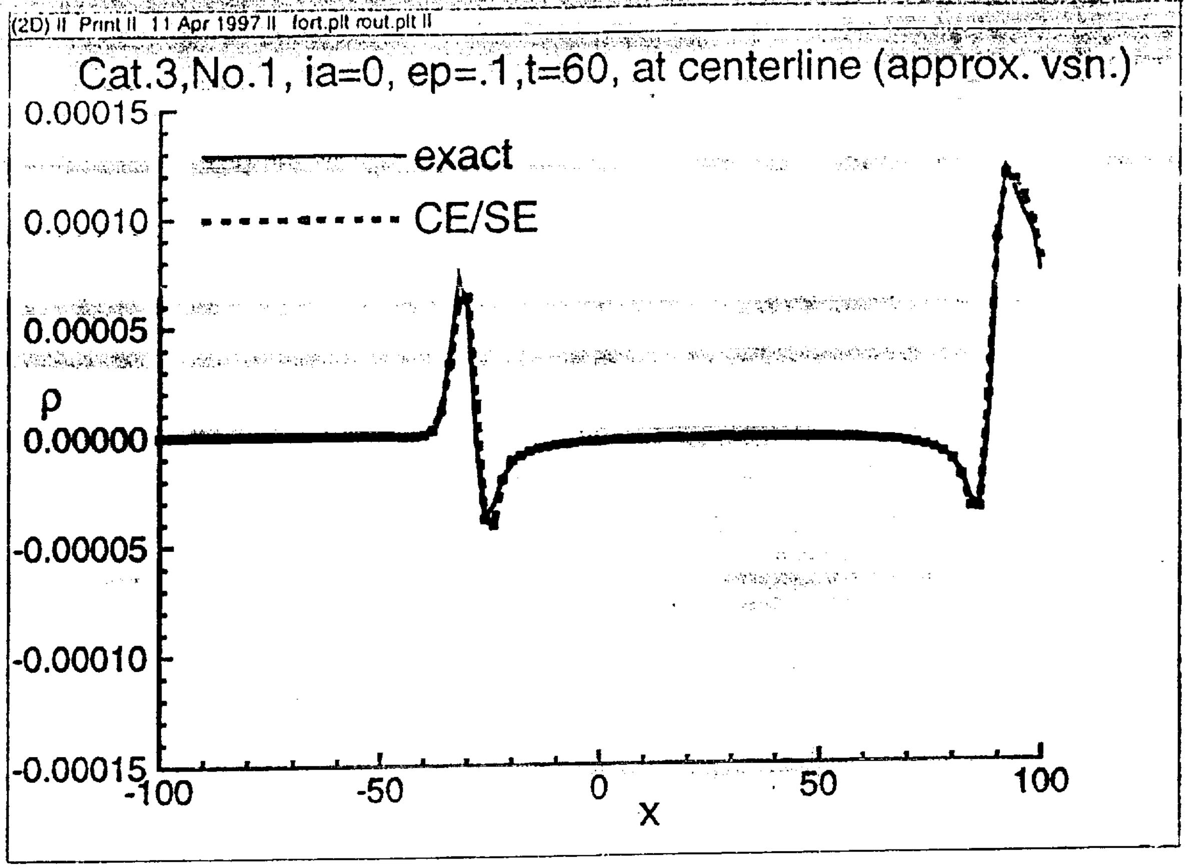
200x20 domain, 200x100 grid
(same contour levels)
— 100x20 domain, 100x100 grid

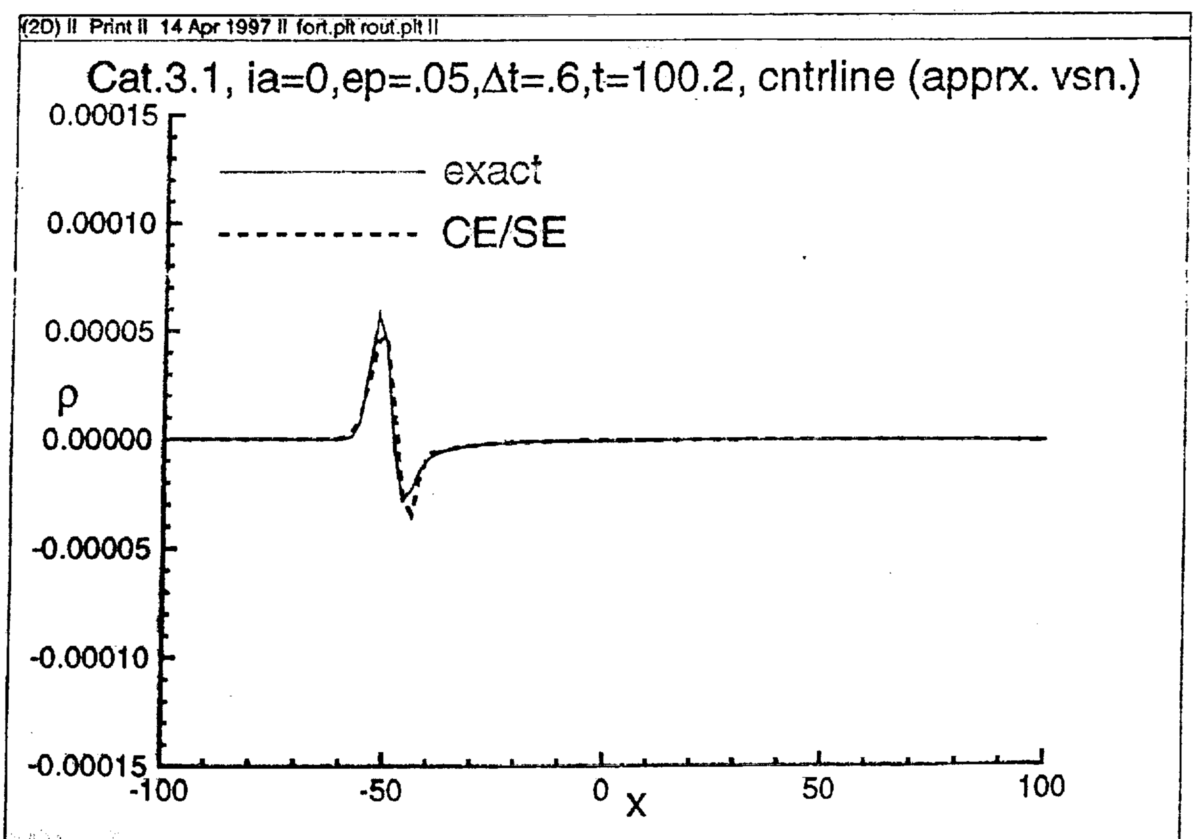
comparison of numerical results with different computational domains, showing effectiveness of outflow NRBC

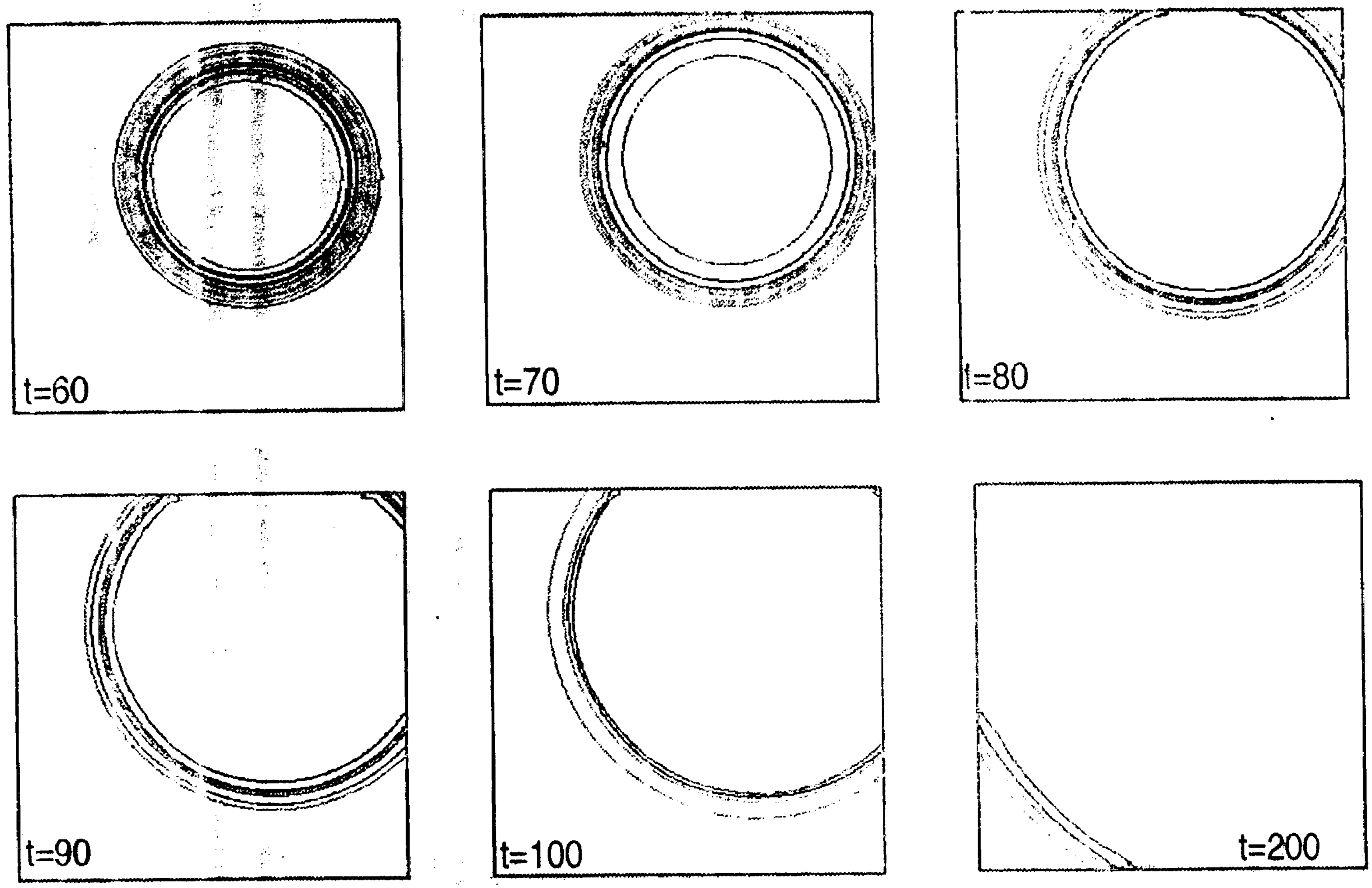
amplitude of eigen functions = 0.02, dt=0.15, 4000 steps



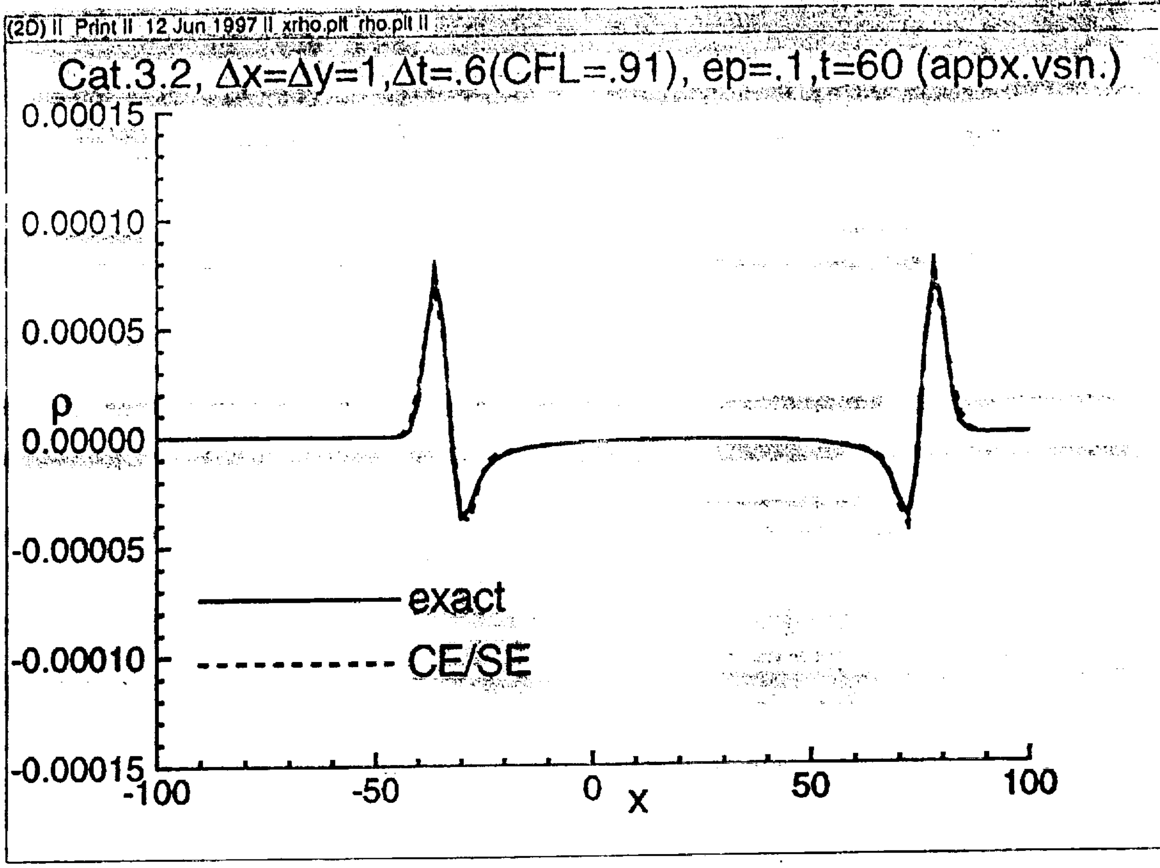
isopycnics at t=30,40,50,60,70,80,100 and 200 (Cat.3.1)


(2D) II Print II 11 Apr 1997 II fort.plt rout.plt II
Cat.3,No.1, ia=0, ep=.1,t=60, at centerline (approx. vsn.)
exact
CE/SE
0.00015
0.00010
0.00005
0.00000
-0.00005
-0.00010
-0.00015
ρ
-100
-50
0
50
100
x

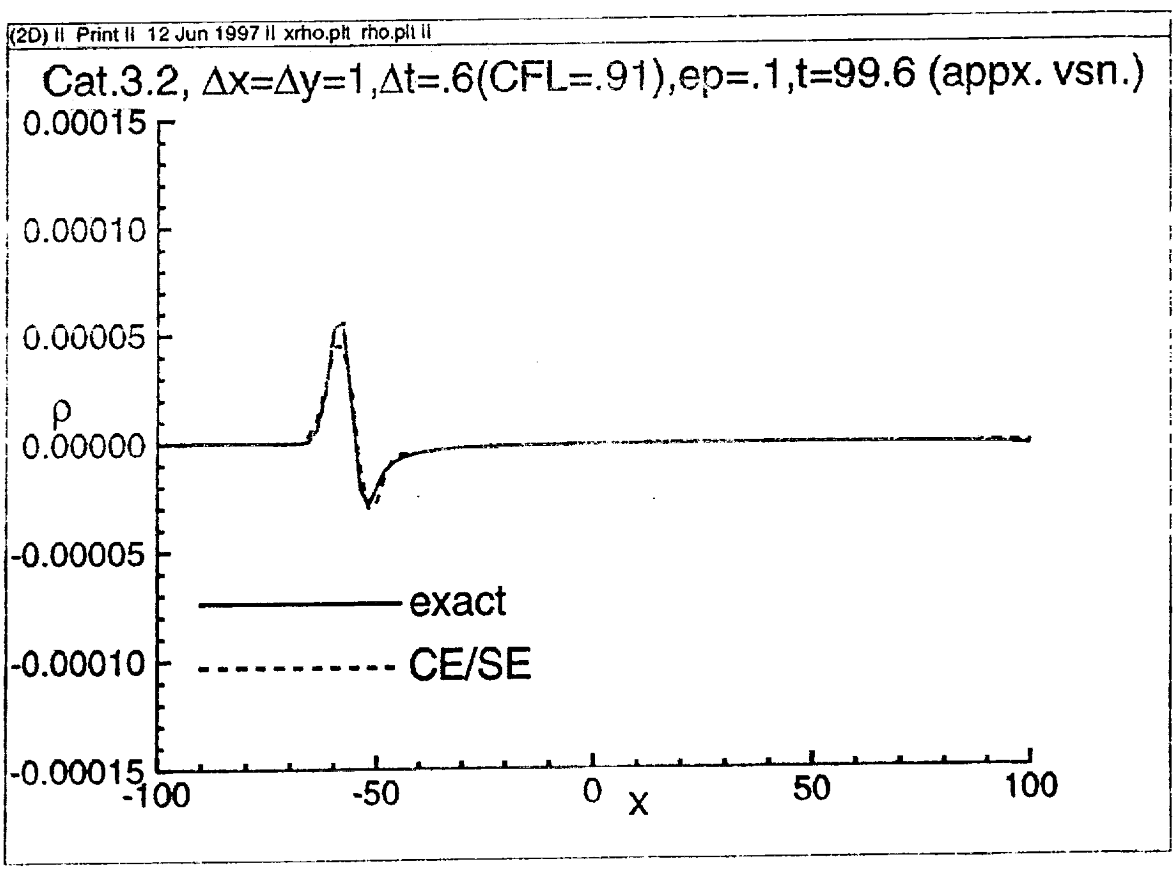

(2D) II Print II 14 Apr 1997 II fort.plt rout.plt II
Cat.3.1, ia=0,ep=.05,Δt=.6,t=100.2, cntrline (apprx. vsn.)
exact
CE/SE
0.00015
0.00010
0.00005
0.00000
-0.00005
-0.00010
-0.00015
ρ
-100
-50
0
50
100
x



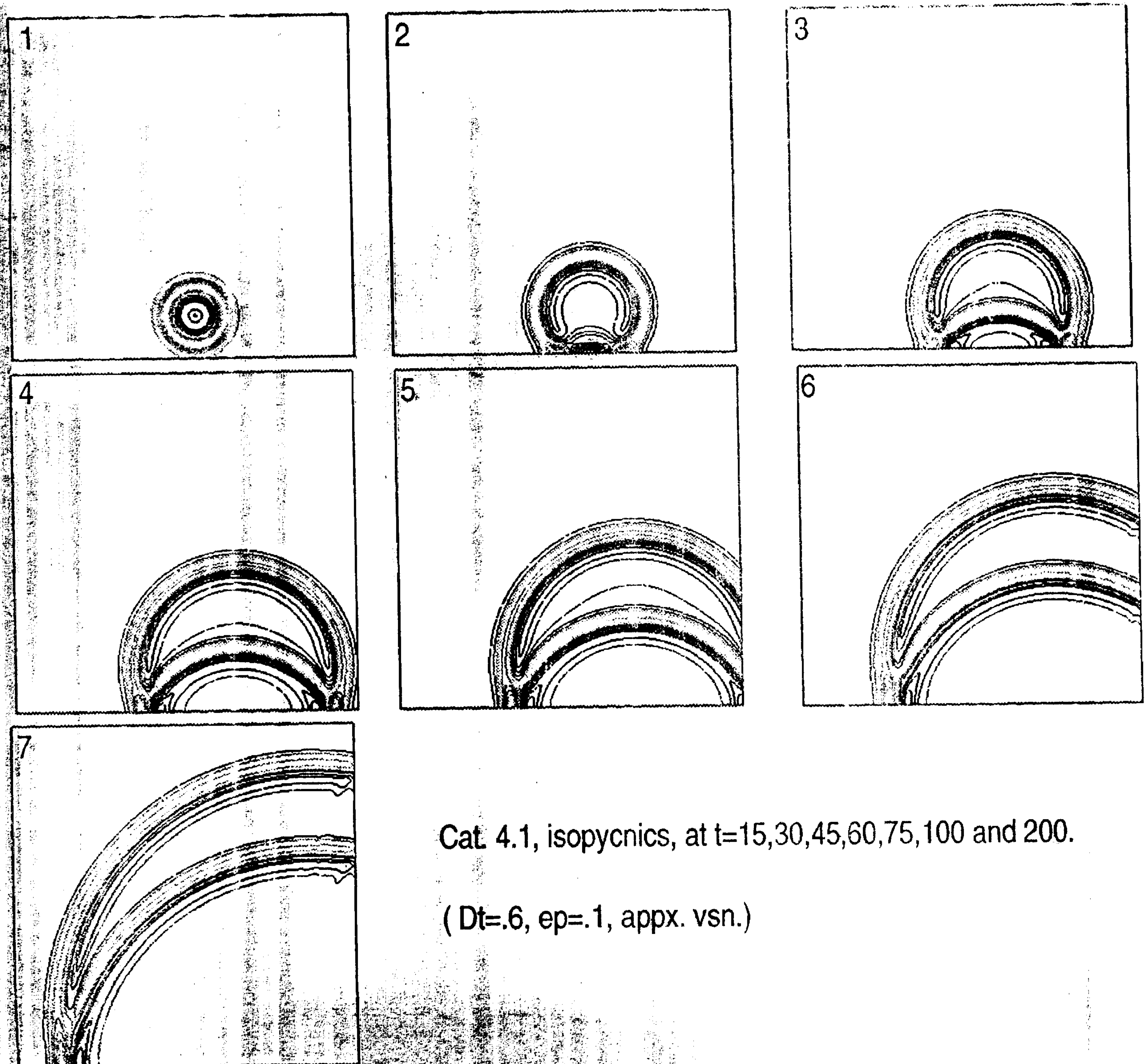
Cat.3.2, pressure at various time .



(2D) II Print II 12 Jun 1997 II xrho.plt rho.plt II
Cat.3.2, Δx=Δy=1,Δt=.6(CFL=.91), ep=.1,t=60 (appx.vsn.)
0.00015
0.00010
0.00005
ρ
0.00000
-0.00005
-0.00010
-0.00015
-100
-50
0
x
50
100
exact
CE/SE

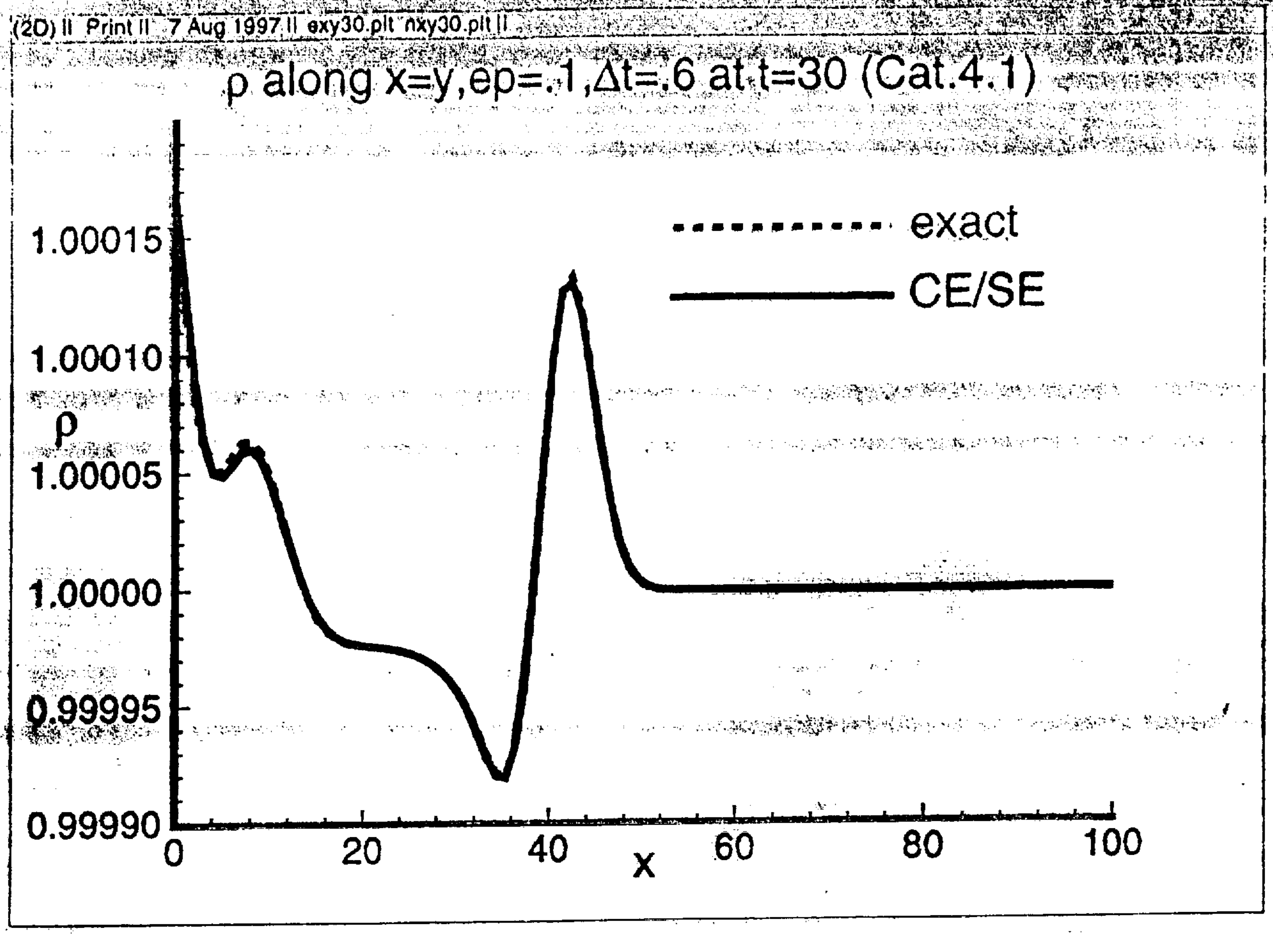


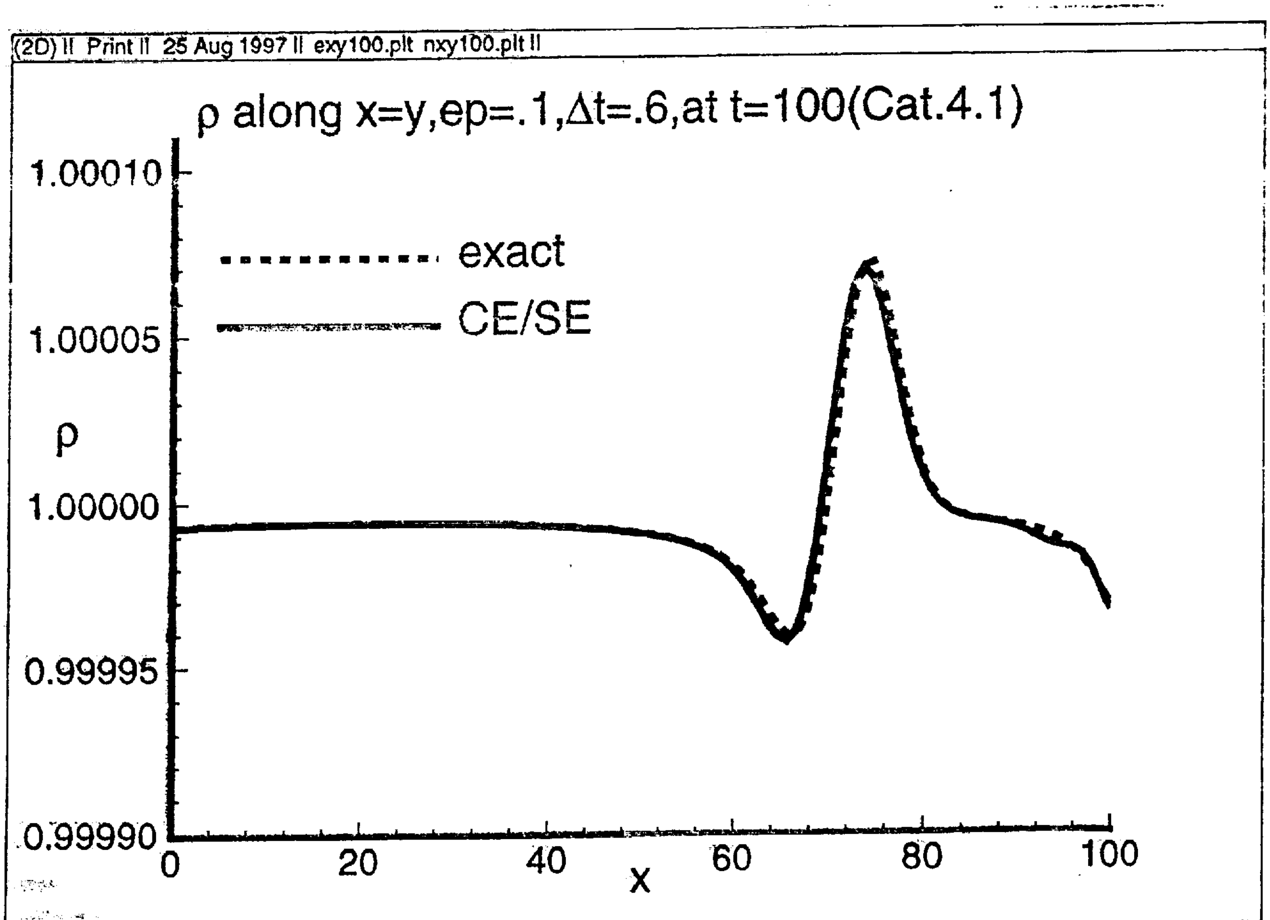
(2D) II Print II 12 Jun 1997 II xrho.plt rho.plt II
Cat.3.2, Δx=Δy=1,Δt=.6(CFL=.91),ep=.1,t=99.6 (appx. vsn.)
0.00015
0.00010
0.00005
ρ
0.00000
-0.00005
-0.00010
-0.00015
-100
-50
0
x
50
100
exact
CE/SE



Cat. 4.1, isopycnics, at t=15,30,45,60,75,100 and 200.

(Dt=.6, ep=.1, appx. vsn.)


(2D) II Print II 7 Aug 1997 II exy30.plt nxy30.plt II
ρ along x=y,ep=.1,Δt=.6 at t=30 (Cat.4.1)
exact
CE/SE
1.00015
1.00010
ρ
1.00005
1.00000
0.99995
0.99990
0
20
40
x
60
80
100


(2D) II Print II 25 Aug 1997 II exy100.plt nxy100.plt II
ρ along x=y,ep=.1,Δt=.6,at t=100(Cat.4.1)
1.00010
exact
CE/SE
1.00005
ρ
1.00000
0.99995
0.99990
0
20
40
x
60
80
100

CAA workshop II, Category 3, probl. 1

(by Wang et al.)

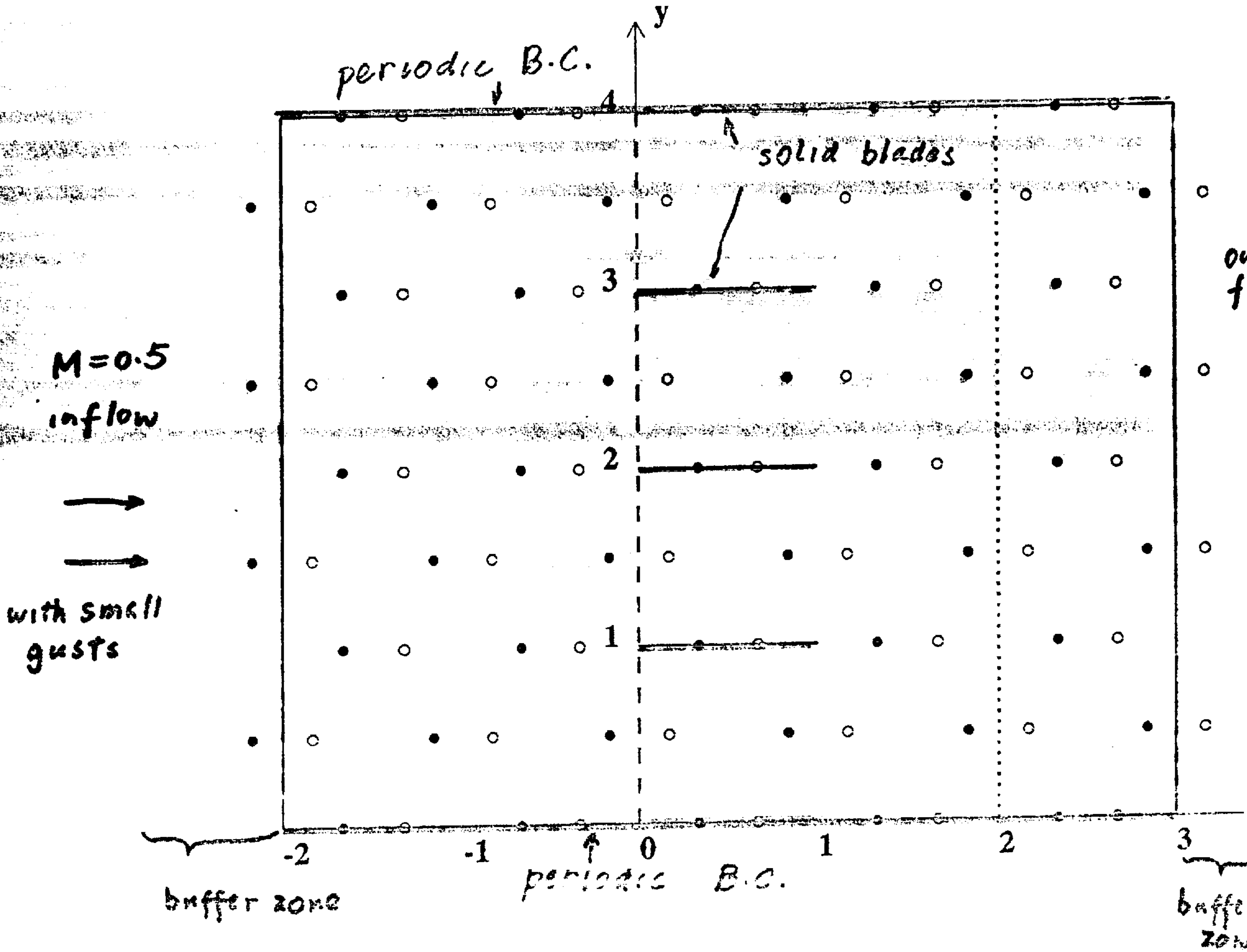
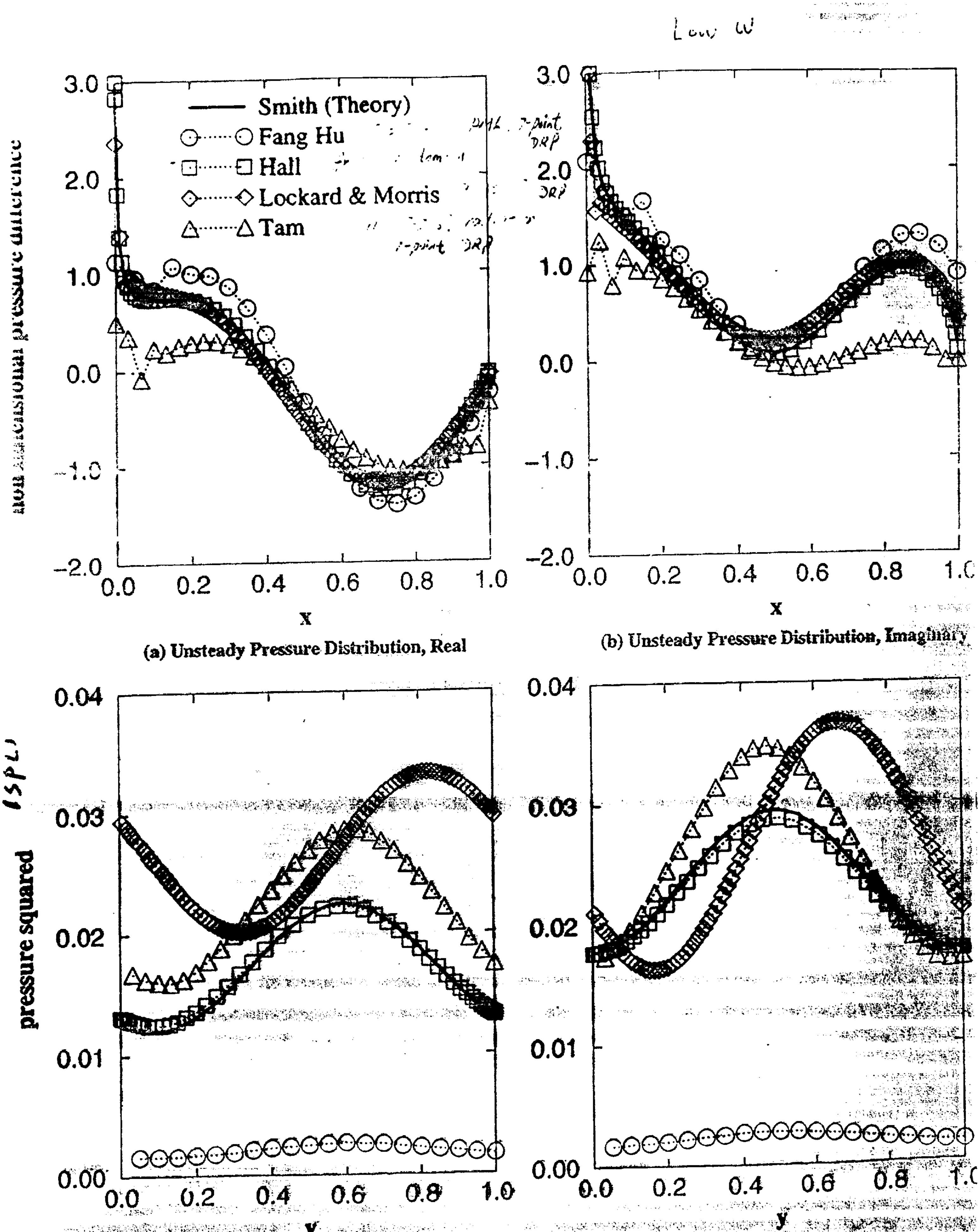


Figure 1: Schematic sketch of computational domain and mesh used in the computation.

(gusts interaction with turbomachine stator blades)



(a) Unsteady Pressure Distribution, Real

(b) Unsteady Pressure Distribution, Imaginary

(c) Upstream Mean Square Pressure, x=−2

(d) Downstream Mean Square Pressure, x=3

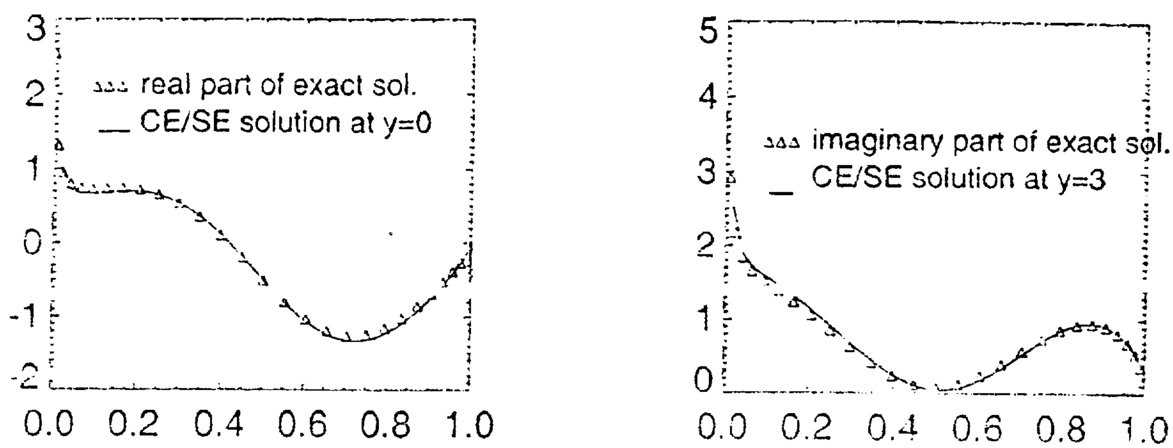


Figure 2: Pressure difference across the airfoil surface compared with the exact solution.

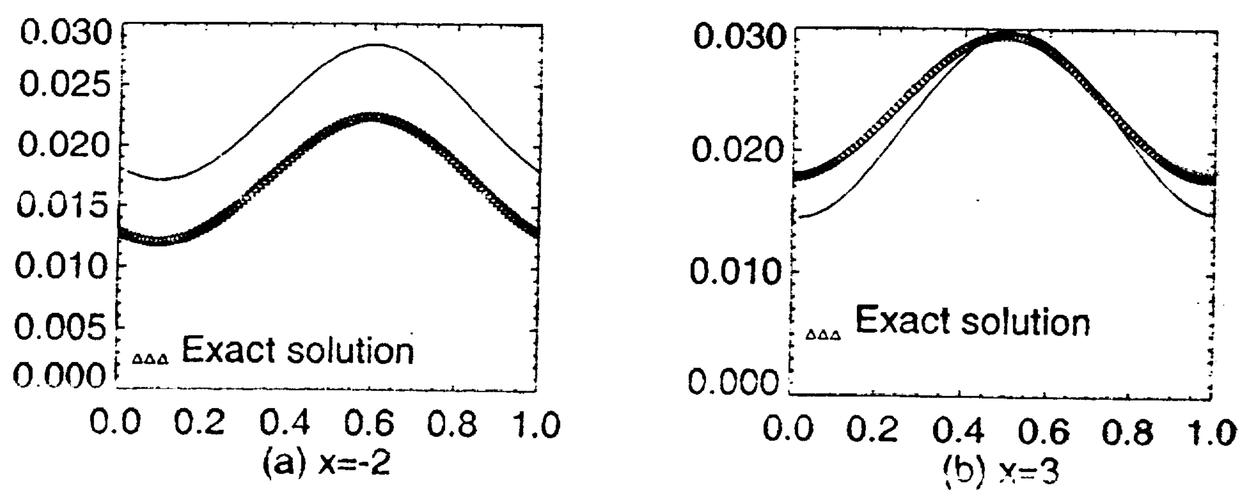
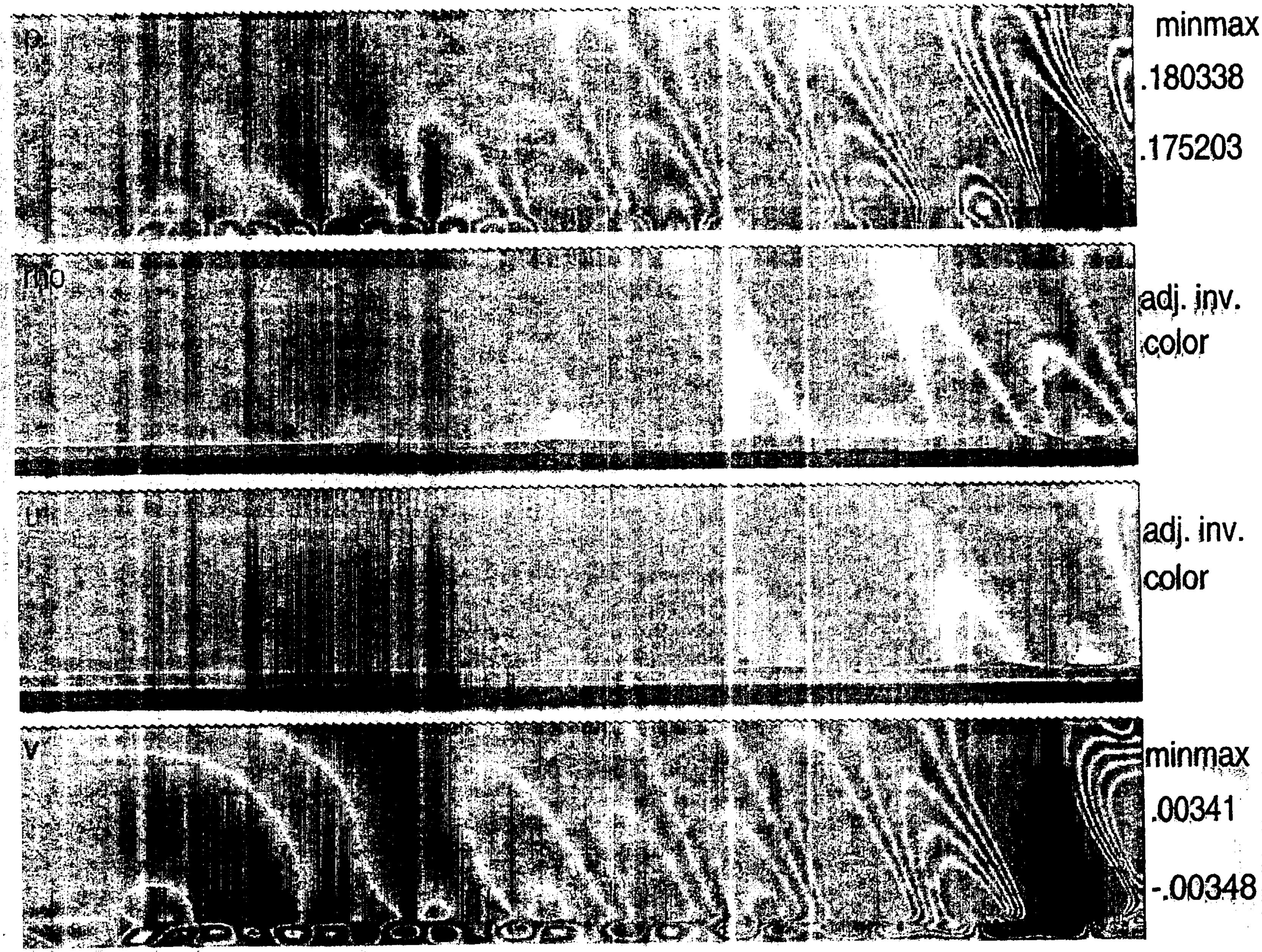
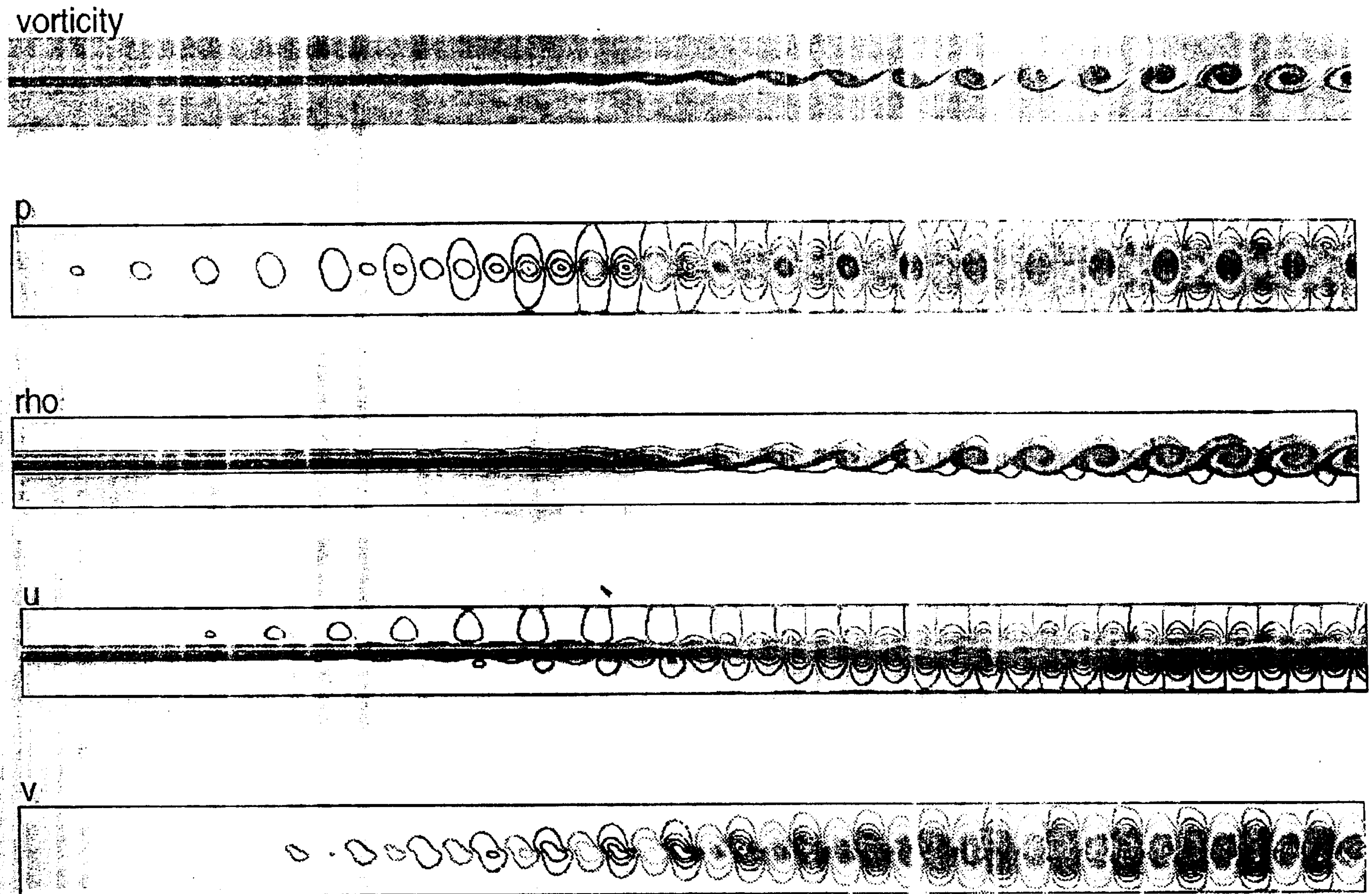


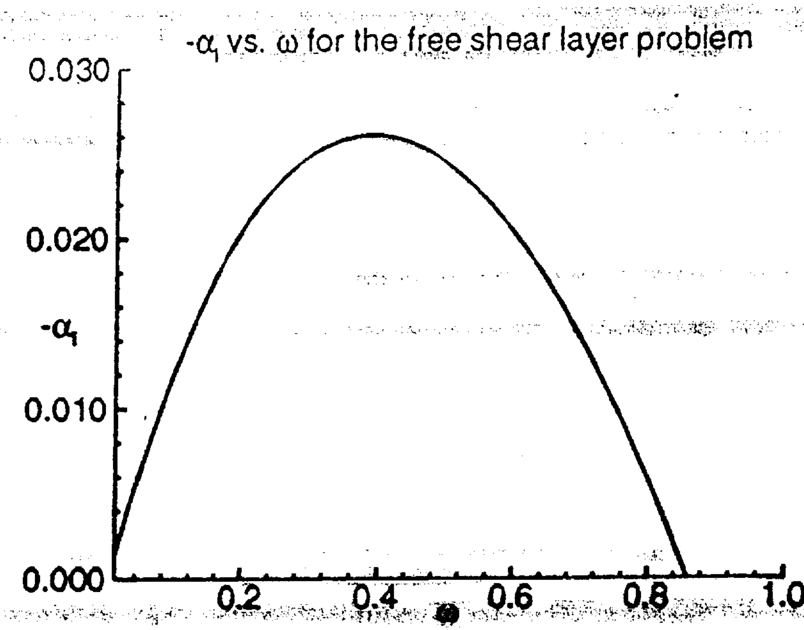
Figure 3: The sound intensity at $x = -2$ and $x = 3$ compared with the exact solution.



Cat.5, snapshot at t=10 (10000 steps), dt=0.001, A=0.001, -5<x<50, 0<y<10
249x140 non-unfm. grid, showing Mach radiation (at 23.5 degree) (St. = 0.14, M=2.0)



1200x200 grid, t=390, dt=.075, 300x20 domain.



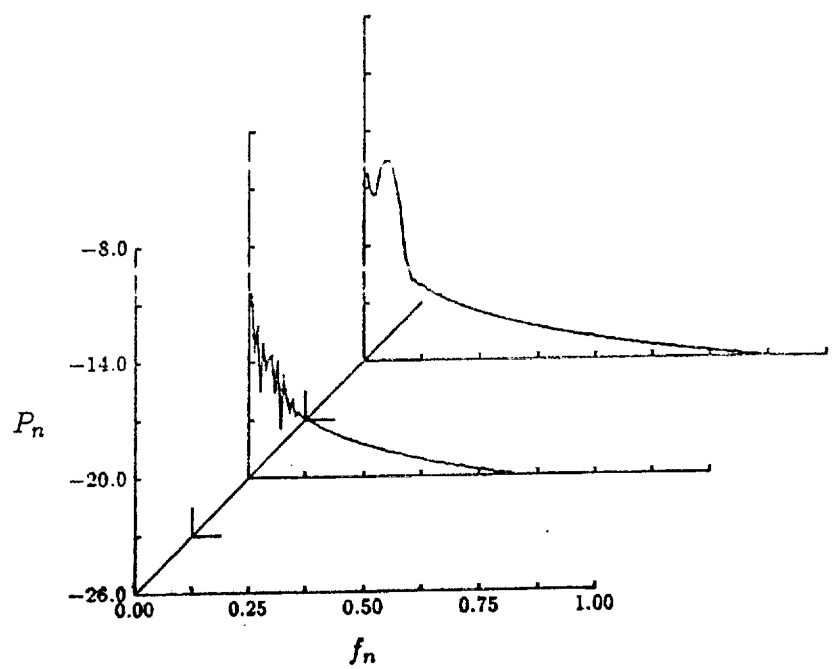


Figure: Power spectra at $x = 150$ and 250 for natural (unforced) case; coarse grid.

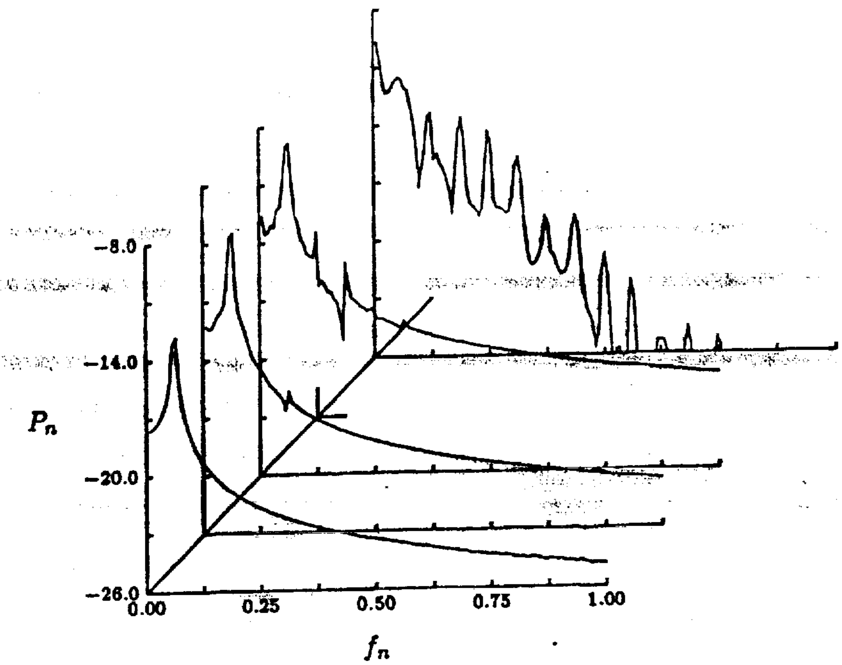


Figure: Power spectra at $x = 050$, 100, 150, and 250 with forcing at the most unstable frequency according to linear theory; coarse grid.

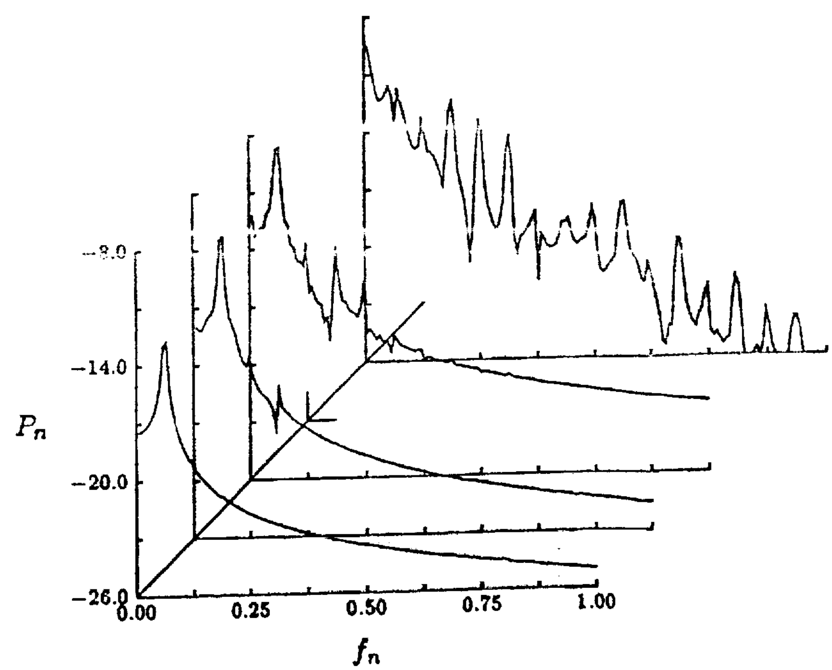


Figure: Power spectra at $x = 050$, 100, 150, and 250 with forcing at most unstable frequency according to linear theory; finer grid.

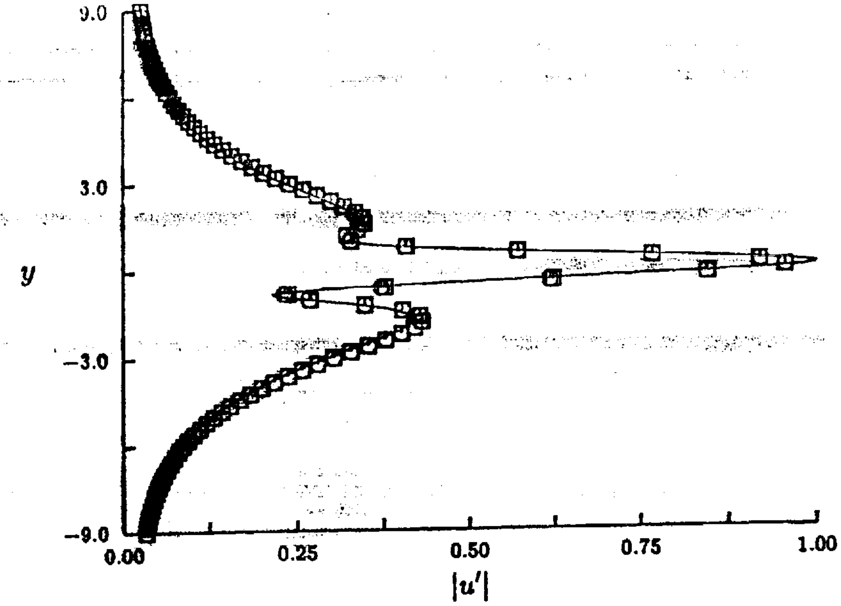


Figure: Transverse mode shape at $x = 100$ with forcing at most unstable frequency acording to linear theory, finer grid. Squares: total u_{rms}; circles: u_{rms} at forcing frequency; solid line: linear eigenfunction (modulus).

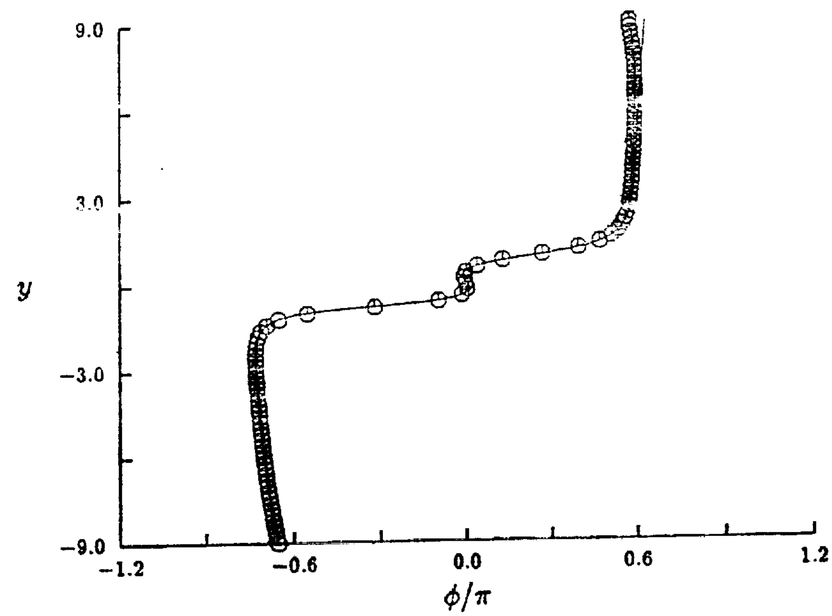


Figure: Transverse phase variation at $x = 100$ with forcing at most unstable frequency acording to linear theory, finer grid. Circles: u_{rms} at forcing frequency; solid line: linear eigenfunction (phase).

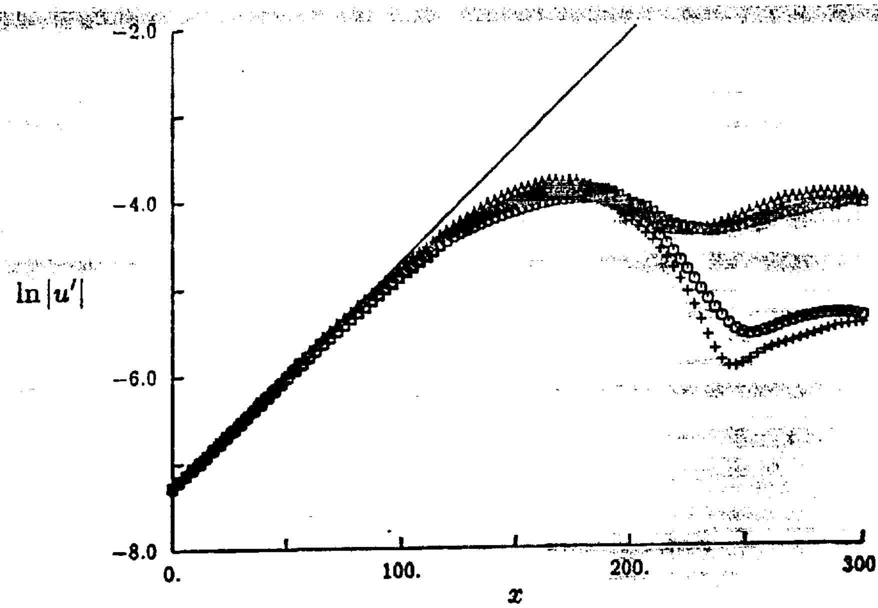


Figure: Streamwise evolution of disturbance amplitude with forcing at the most unstable frequency according to linear theory. Squares: total u_{rms}, coarse grid; triangles: total u_{rms}, finer grid; circles: u_{rms} at forcing frequency, coarse grid; crosses: u_{rms} at forcing frequency, finer grid; solid line: linear growth.

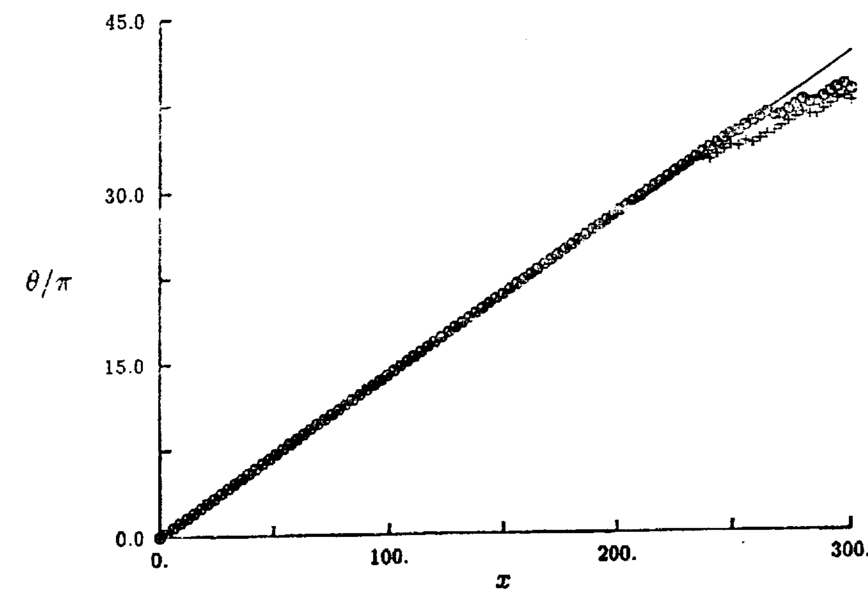
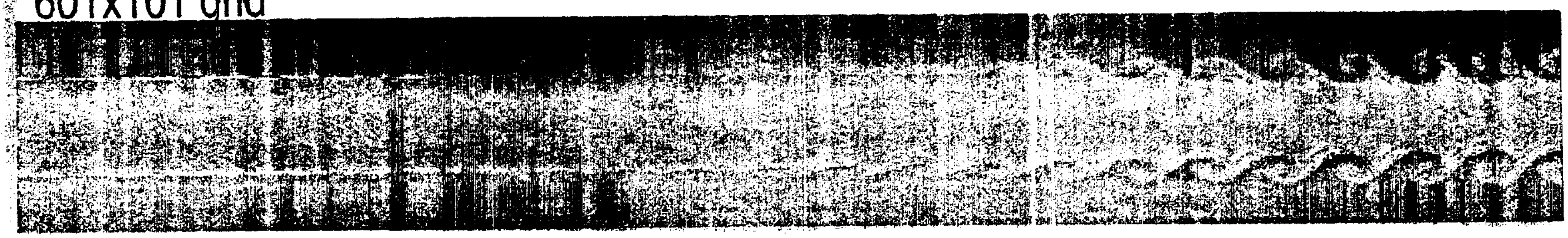
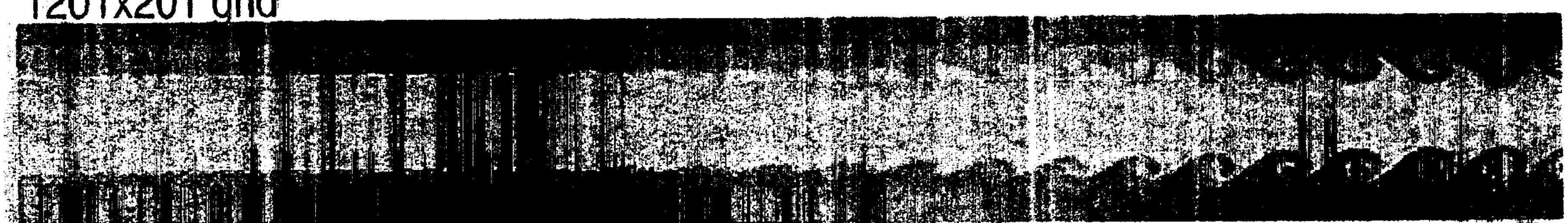


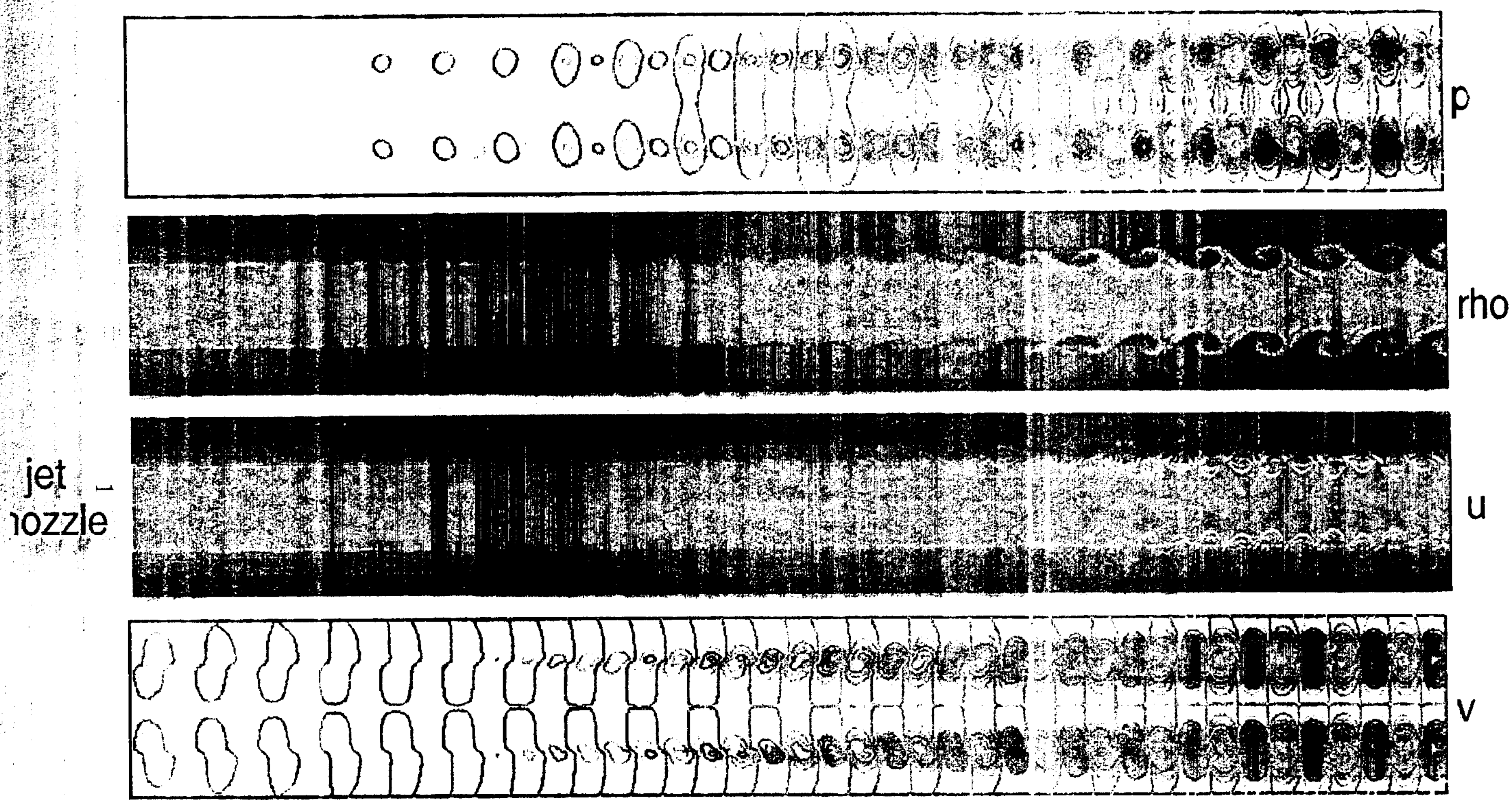
Figure: Streamwise evolution of disturbance phase with forcing at the most unstable frequency according to linear theory. Circles: coarse grid; crosses: finer grid; solid line: linear result.

601x101 grid

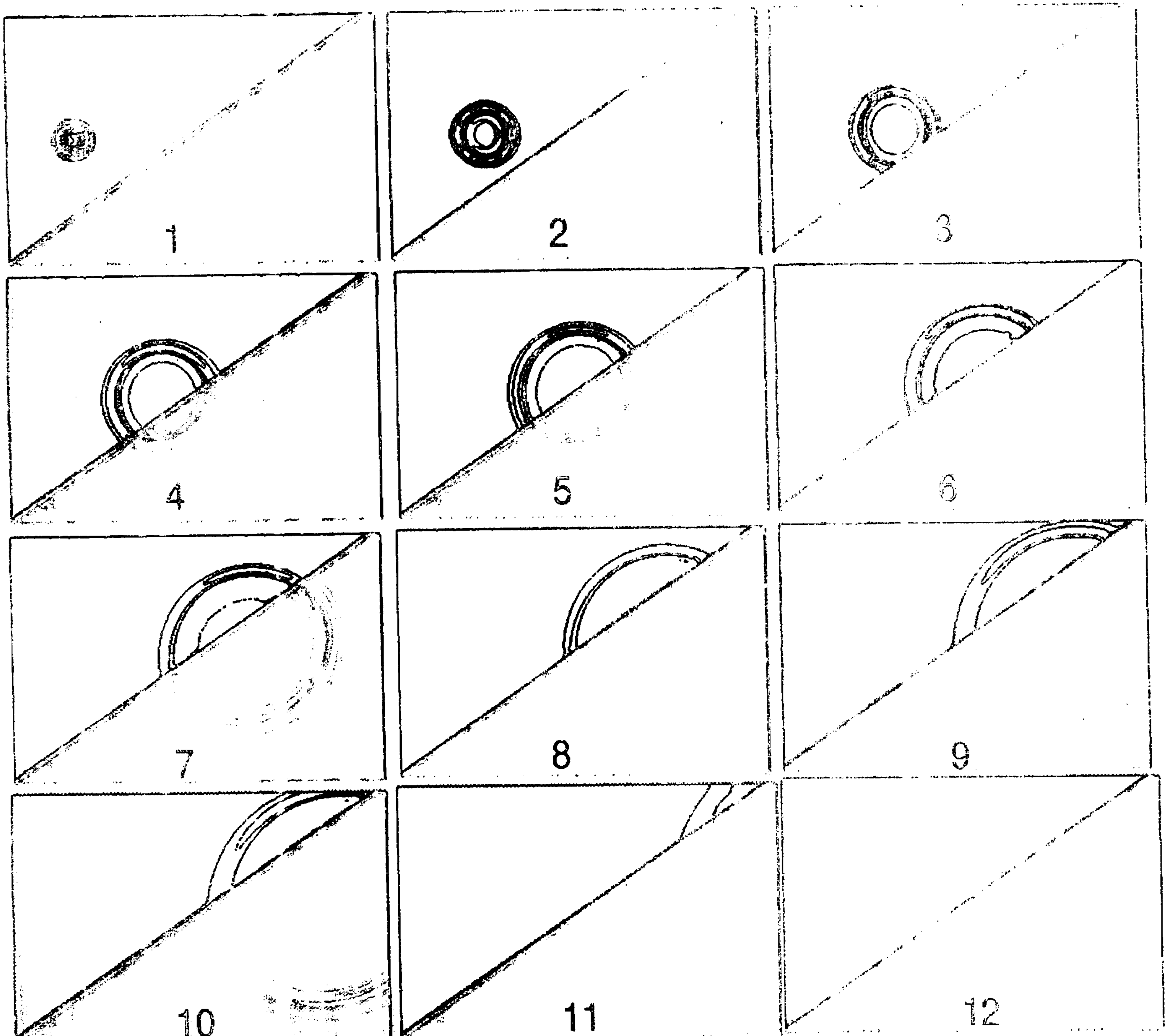


1201x201 grid

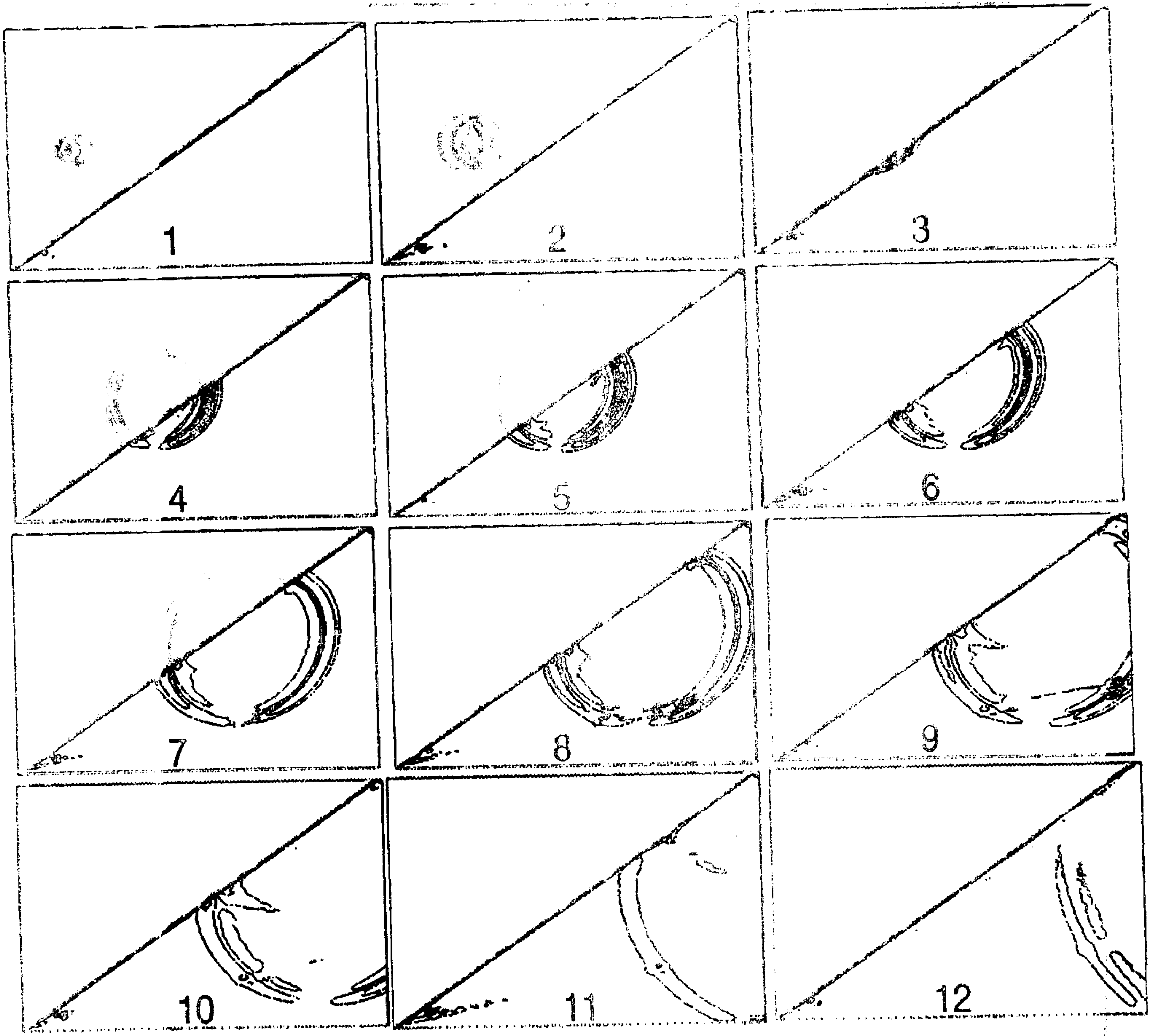




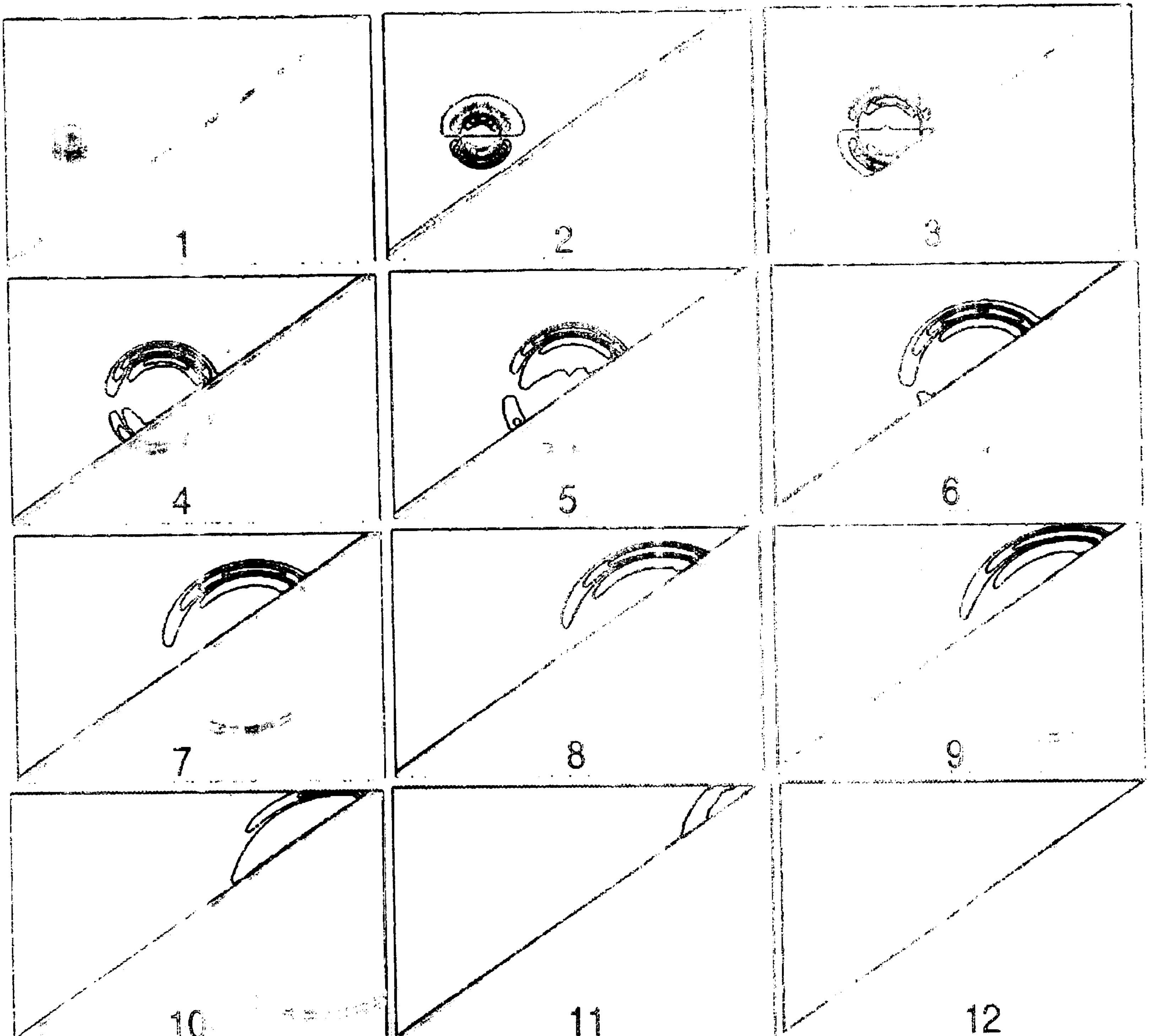
axisymmetric jet instability, showing roll-up ; t=390, 600x100 grid, perturbed with eigenfunctions, 300x20 domain, dt=0.15.



Acoustic pulse passing through a shock (isobars)

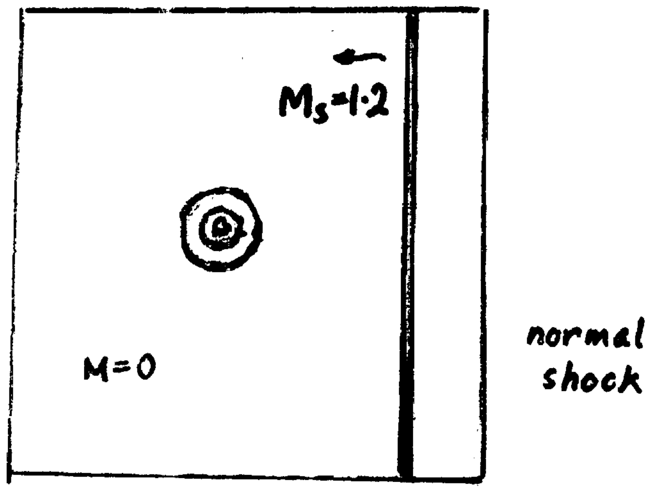


acoustic pulse passing through a shock (u- contours)



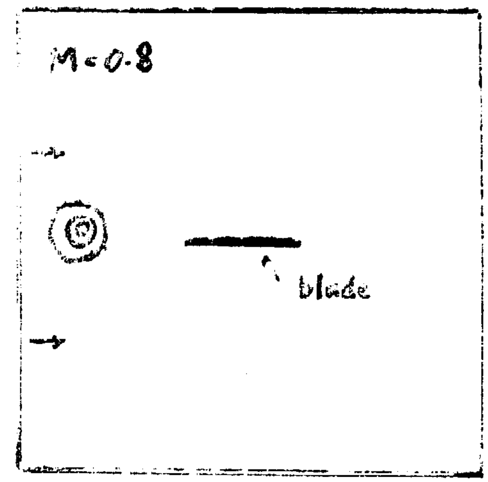
acoustic pulse passing through a shock (v- contours)

1. shock-vortex Interaction



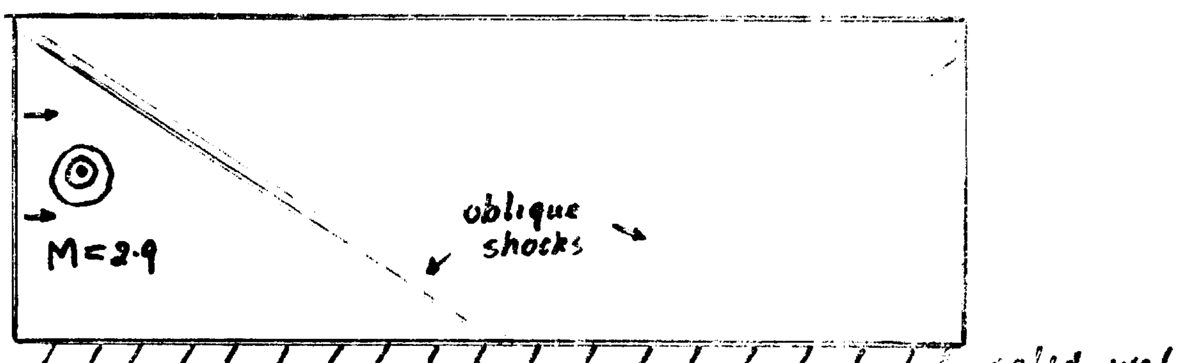
150x150 uniform grid

2. BVI

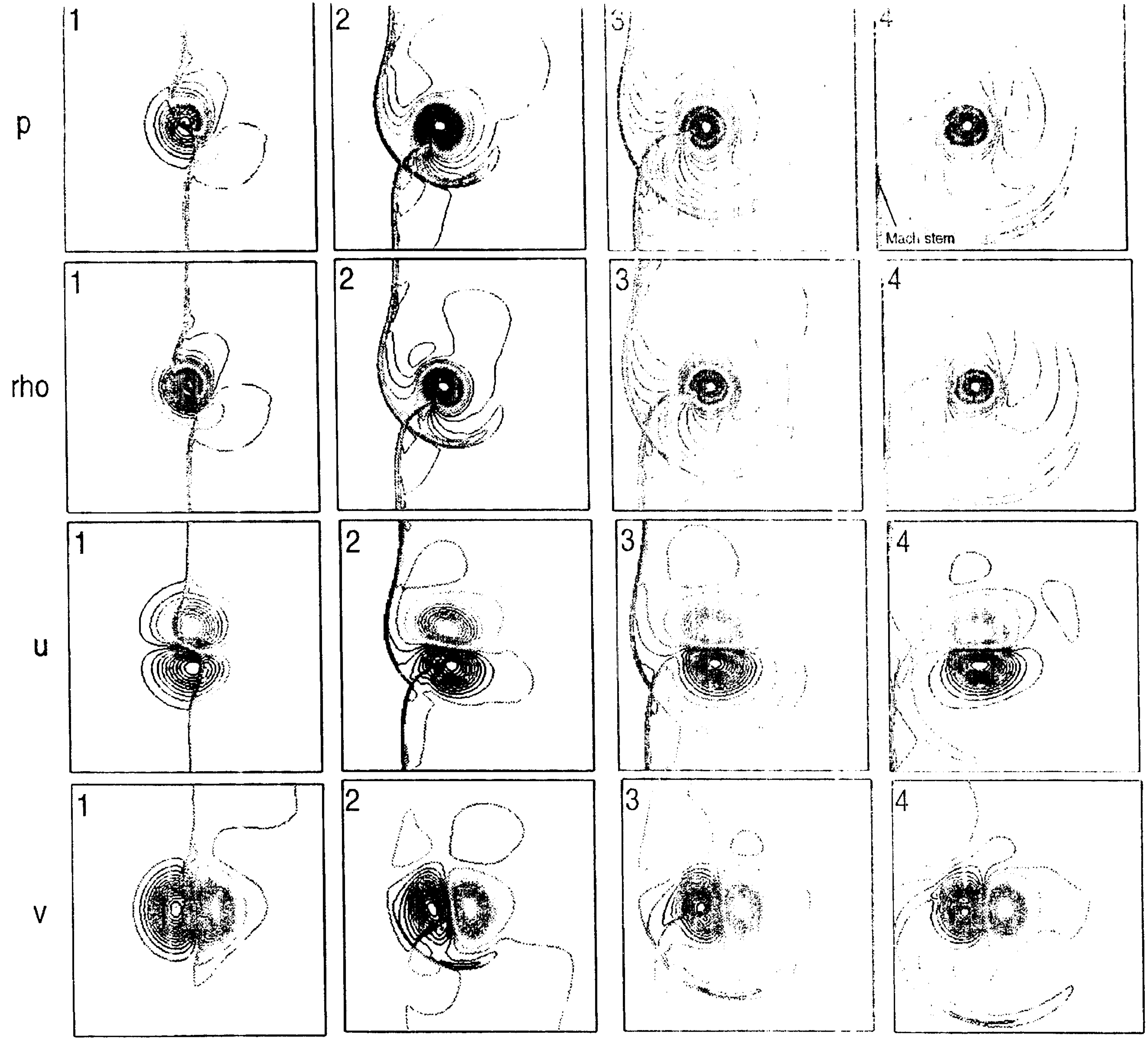


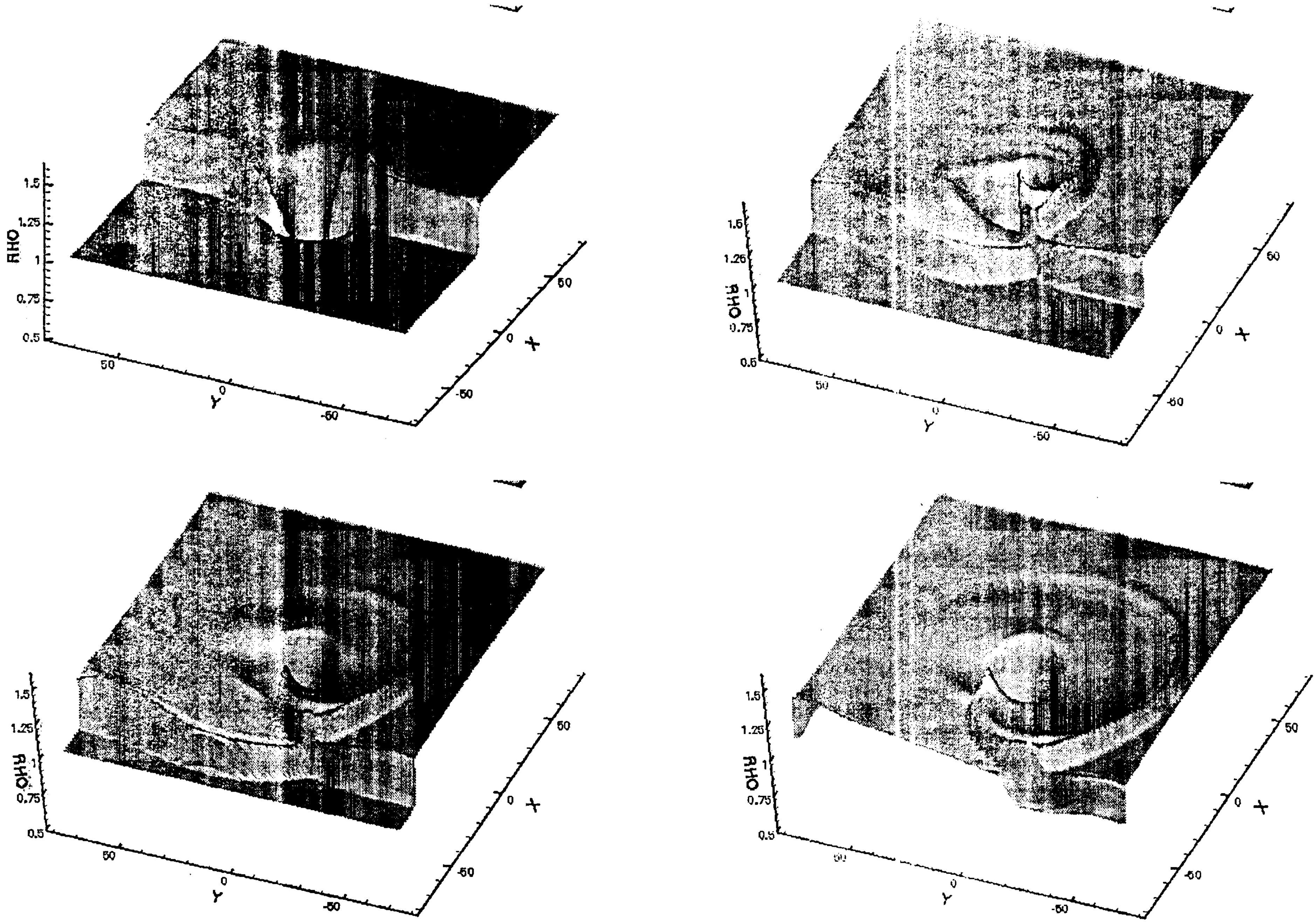
200 x 200 uniform grid

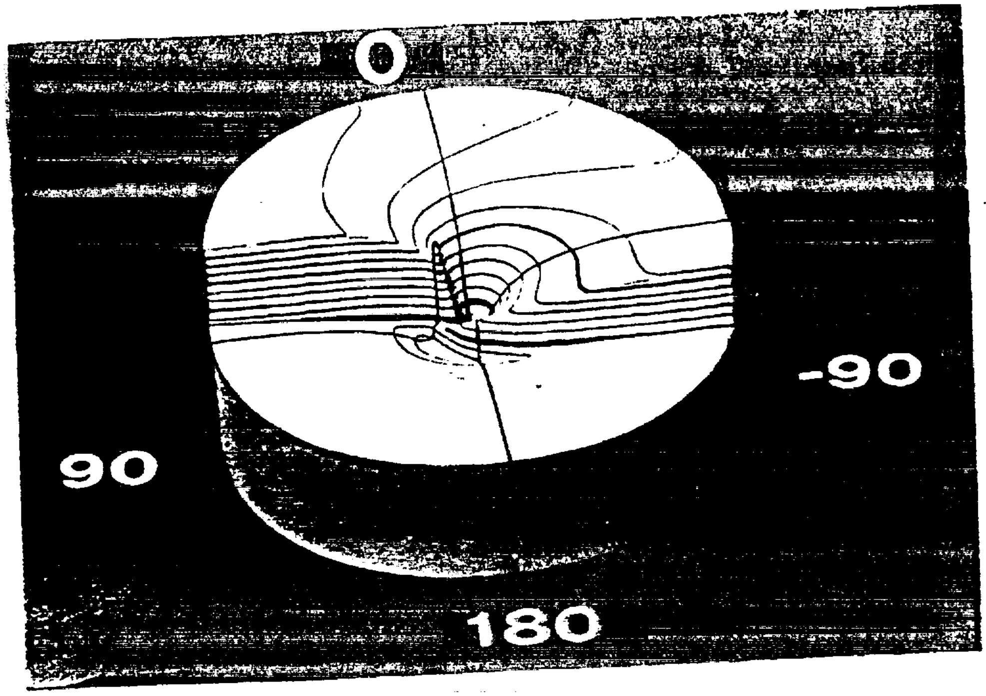
3. multiple vortex-shock interaction



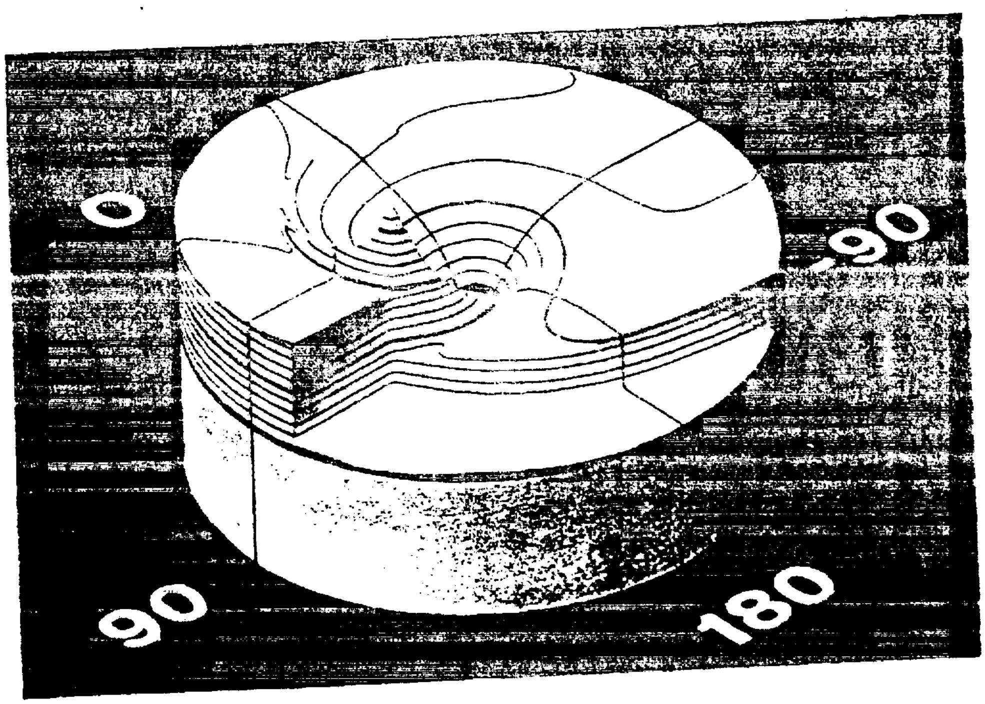
400 x 100 uniform grid


p
rho
u
v
1
2
3
4
Mach stem


RHO
0.5
0.75
1
1.25
1.5
Y
50
0
-50
X
50
0
-50

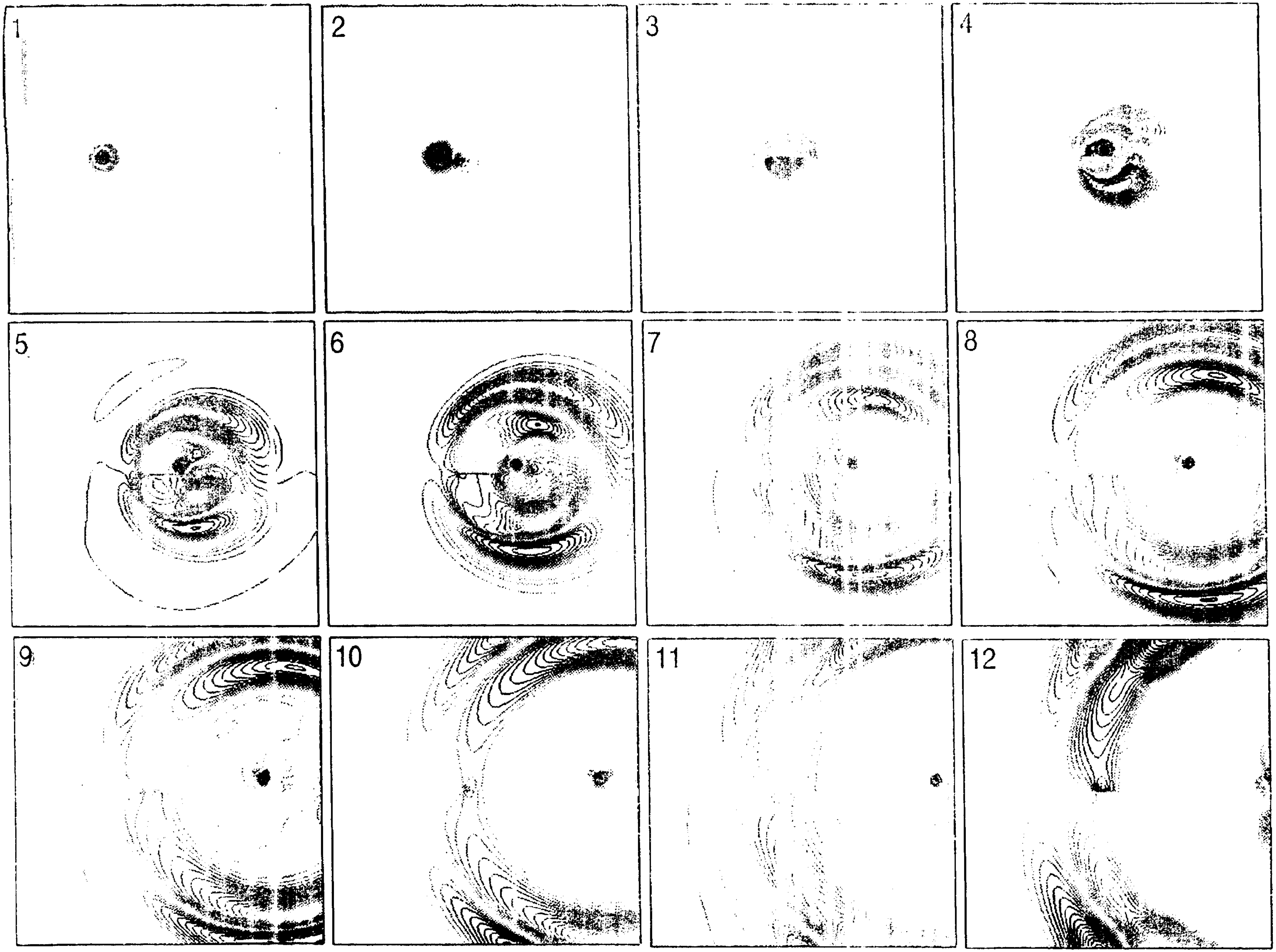


Density relief ρ_2/ρ_1 = 1,27 t = 18 μs

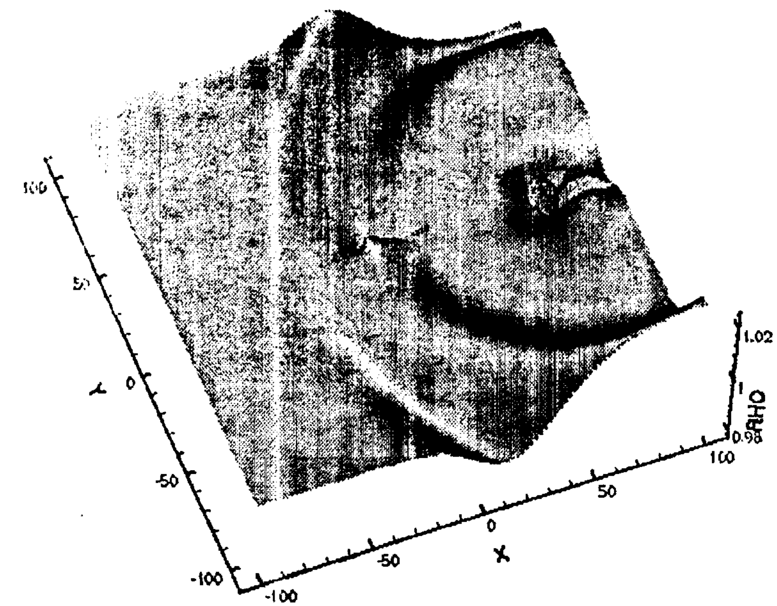


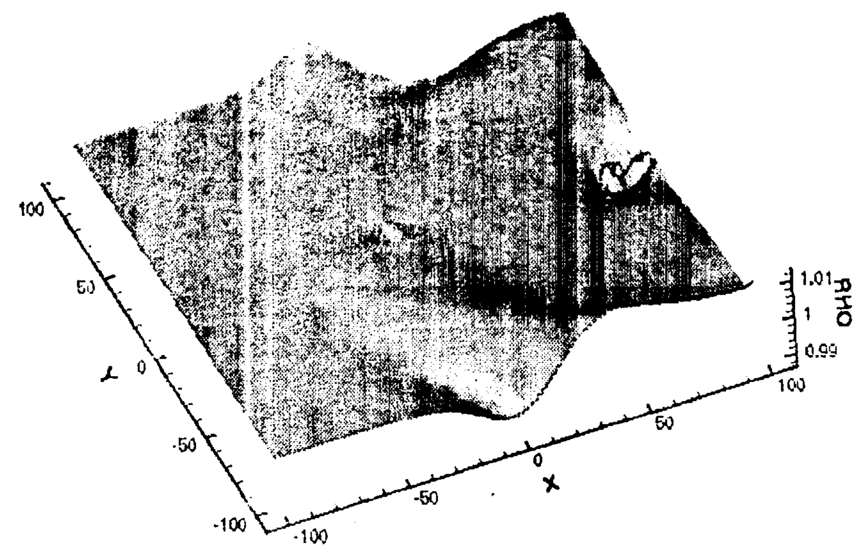
Density relief ρ_2/ρ_1 = 1,27 t = 98 μs

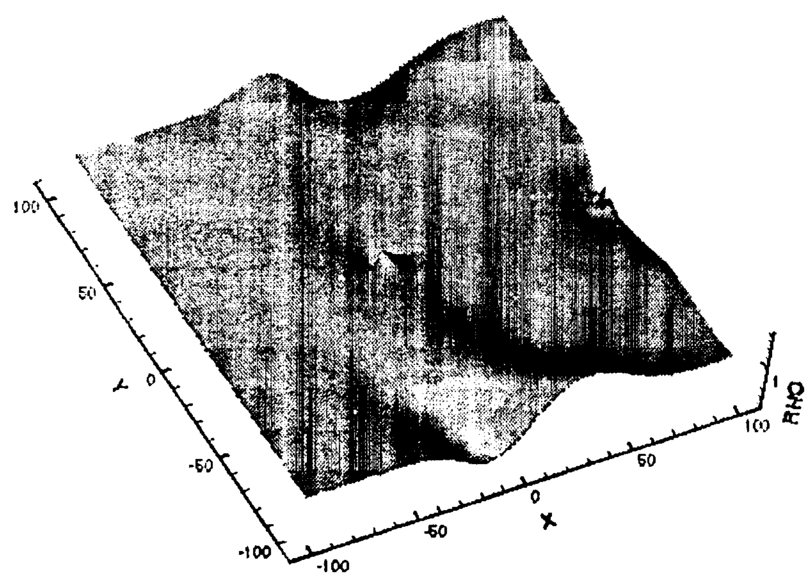
Naumann & Hermanns



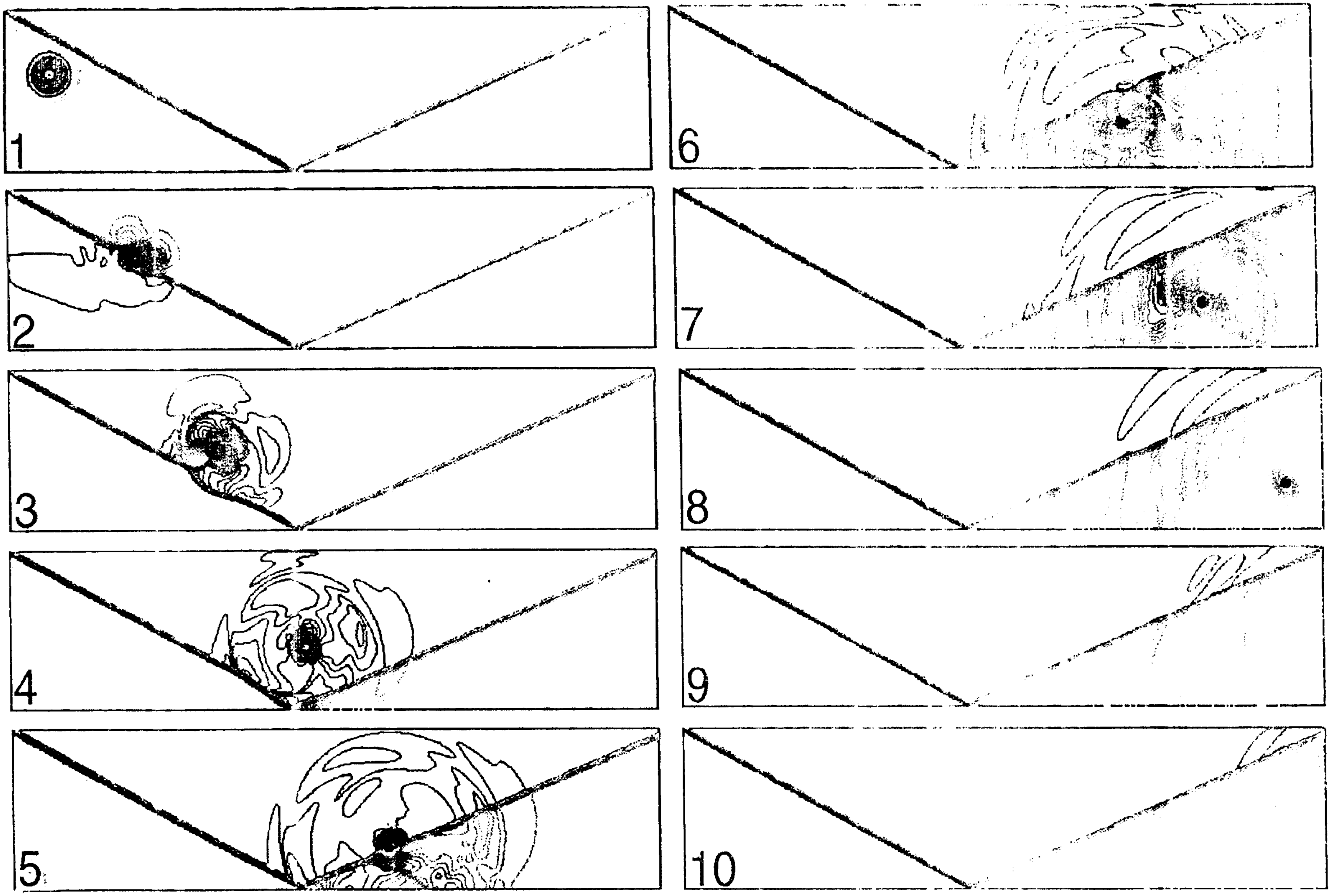
vortex-flat plate interaction (BVI), isobars, 40-480 steps, 200x200 grid.



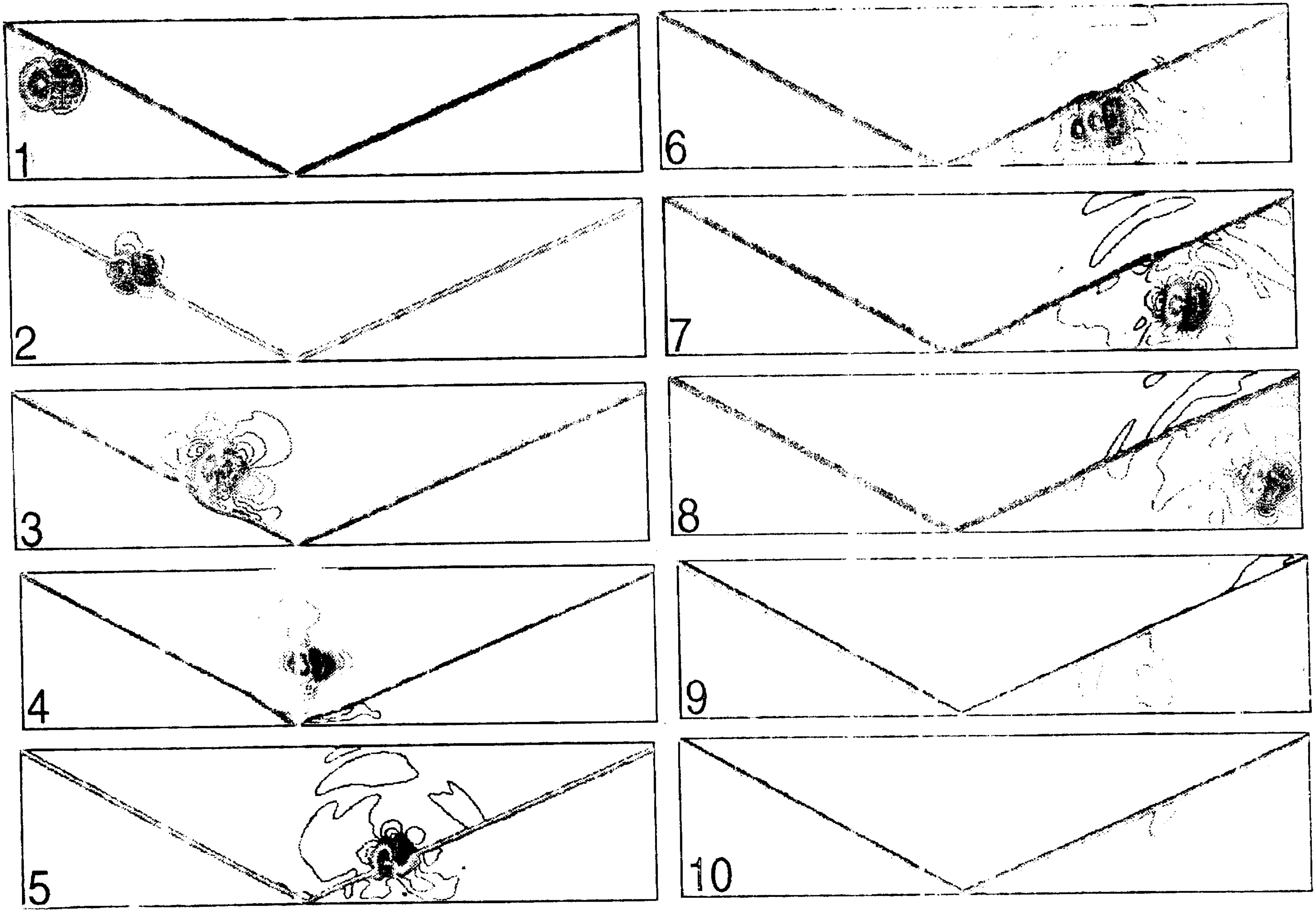




density carpet plots at t= 400,440,480
for theBVI problem

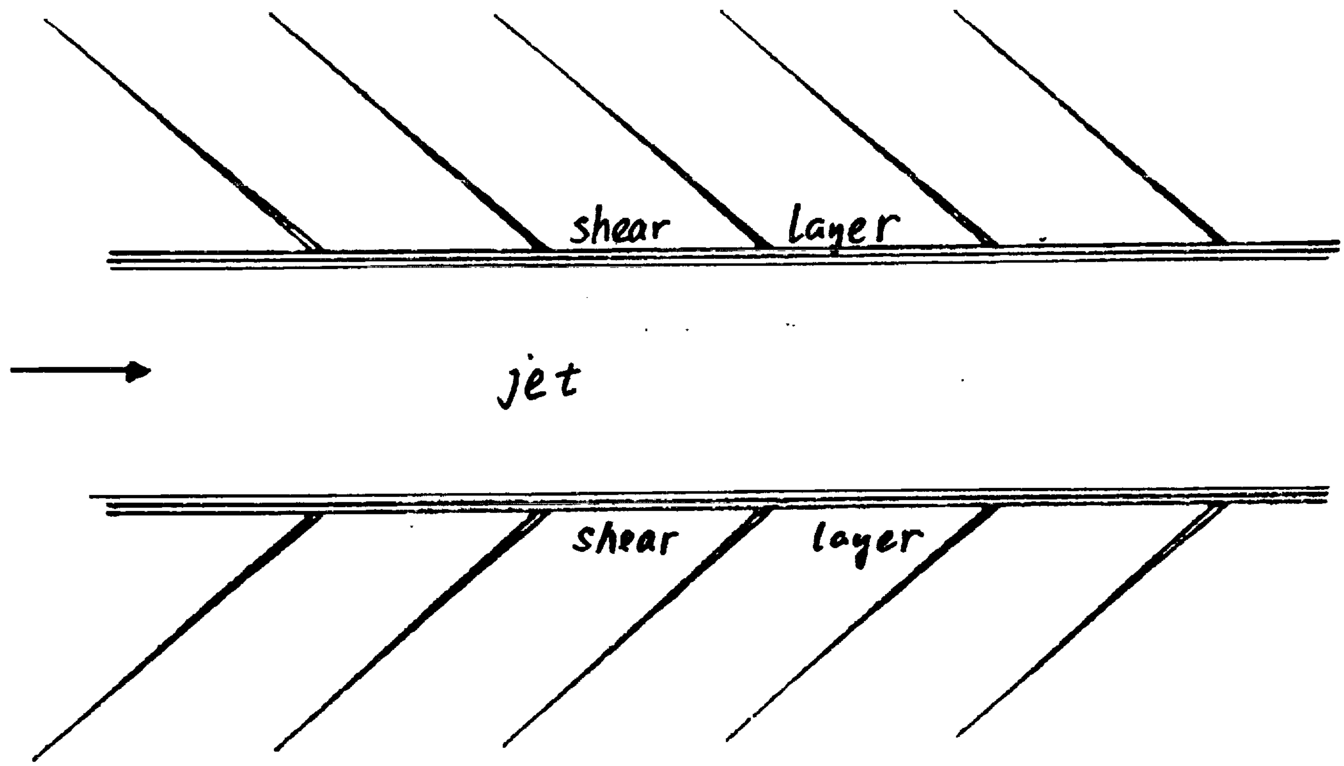


isobars for a vortex passing through shocks, with acoustic waves generated.

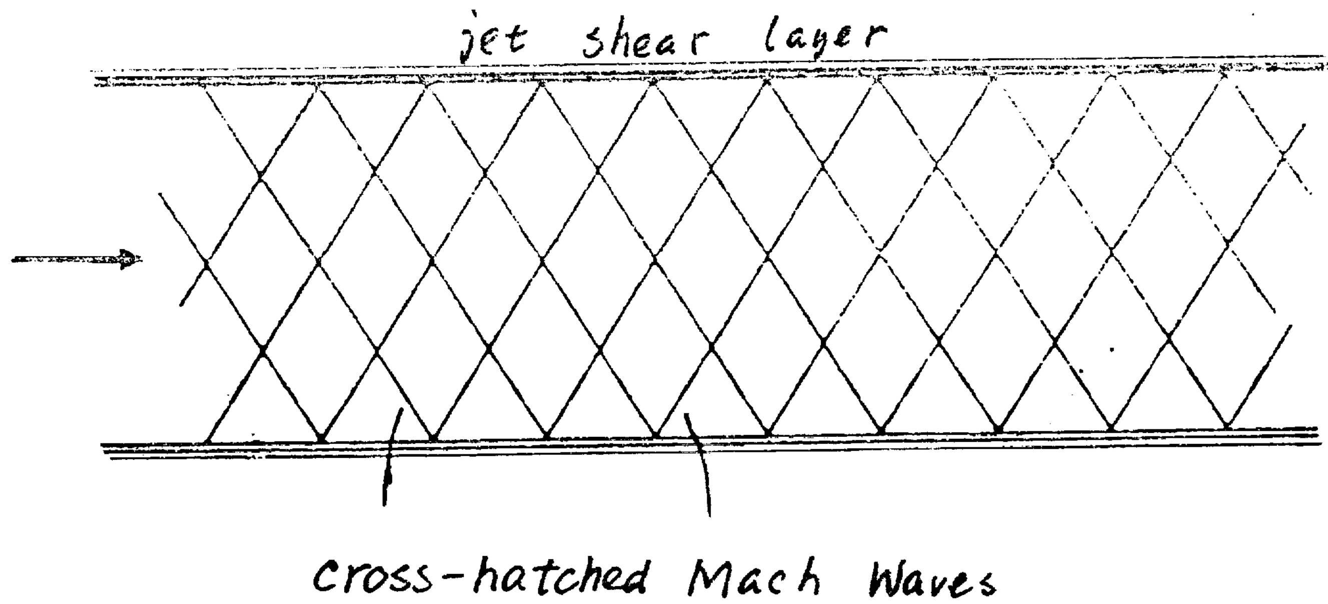


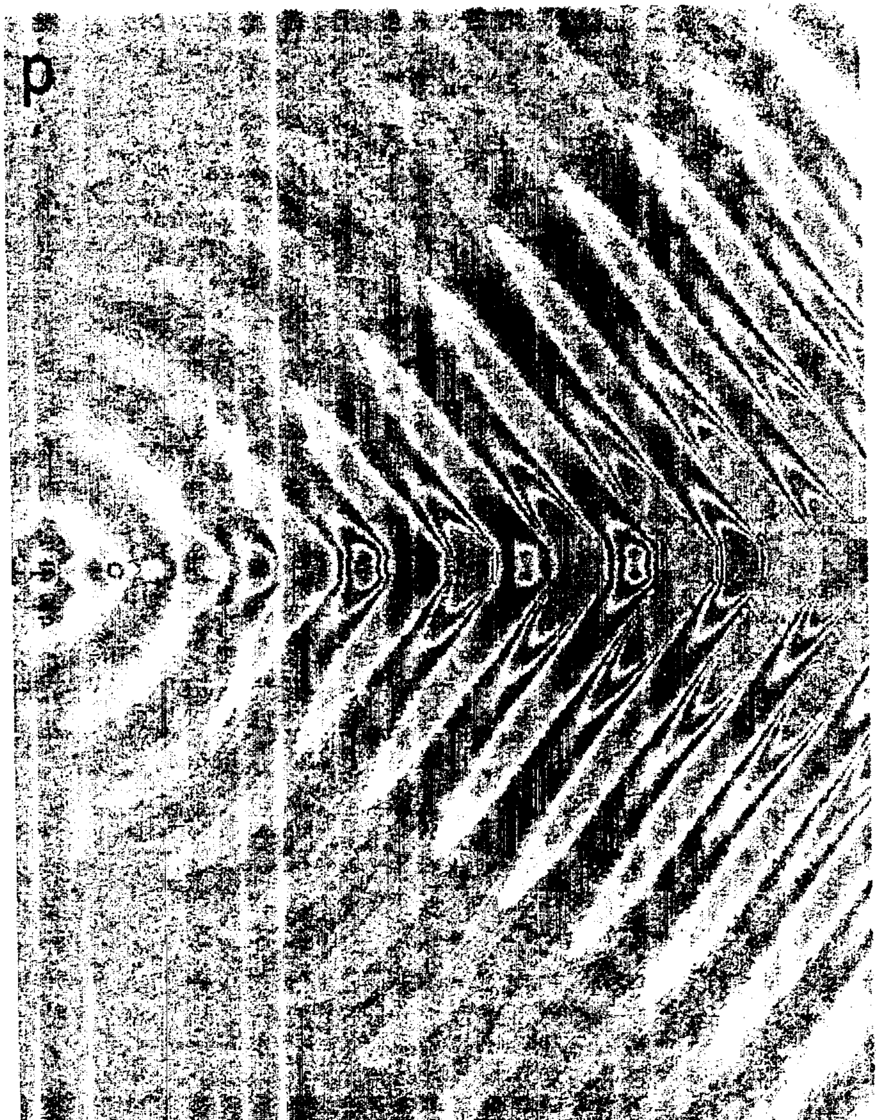
v-contours for a vortex passing through shocks

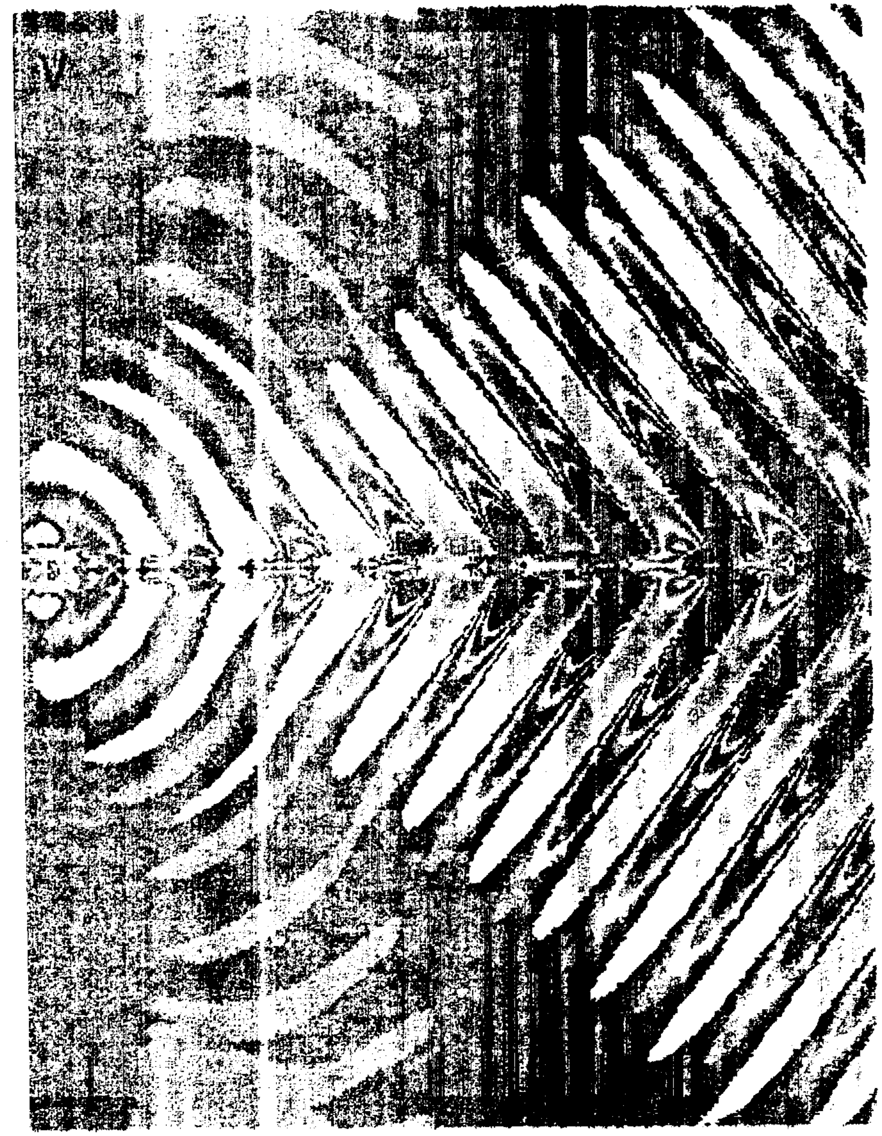
Mach Radiation



Jet as a wave guide

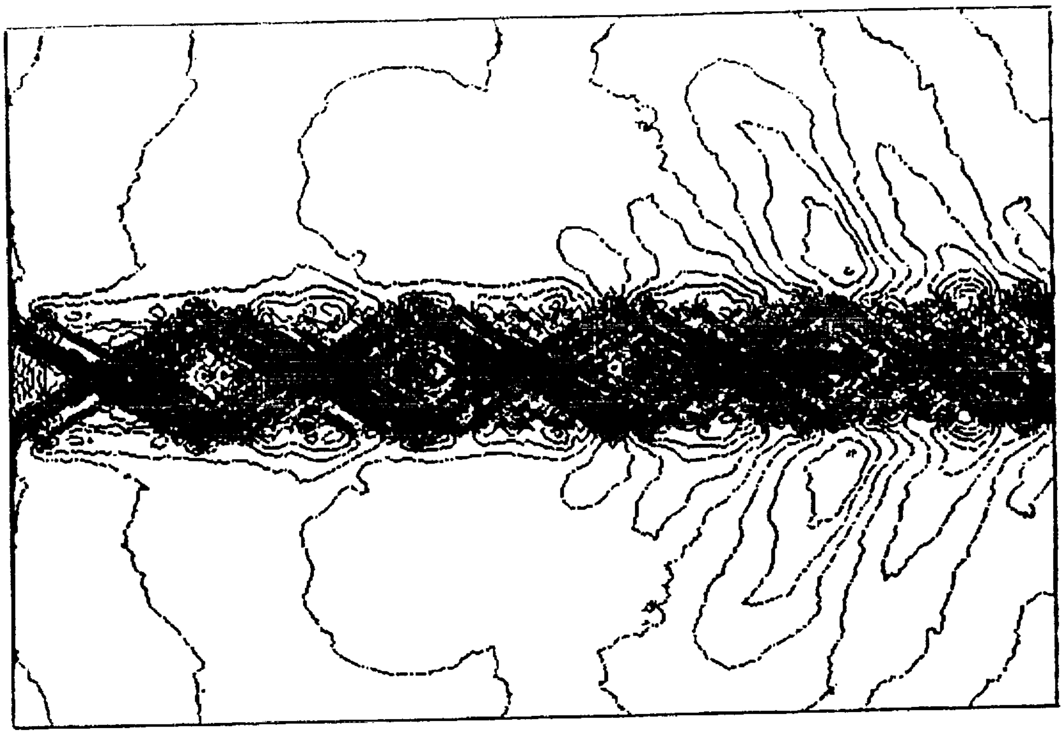


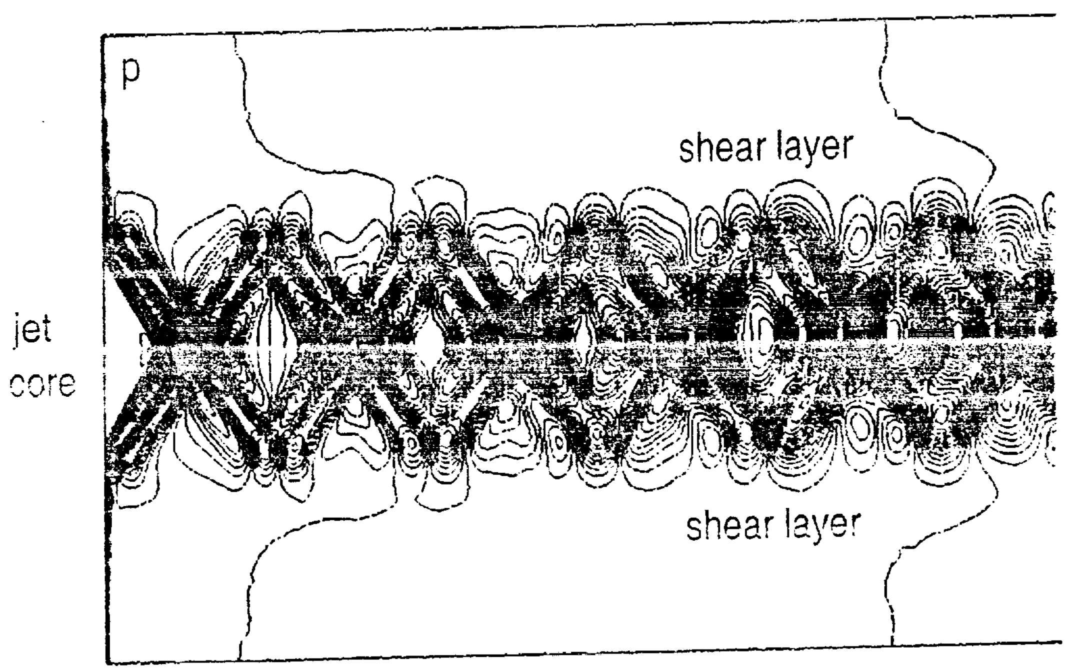




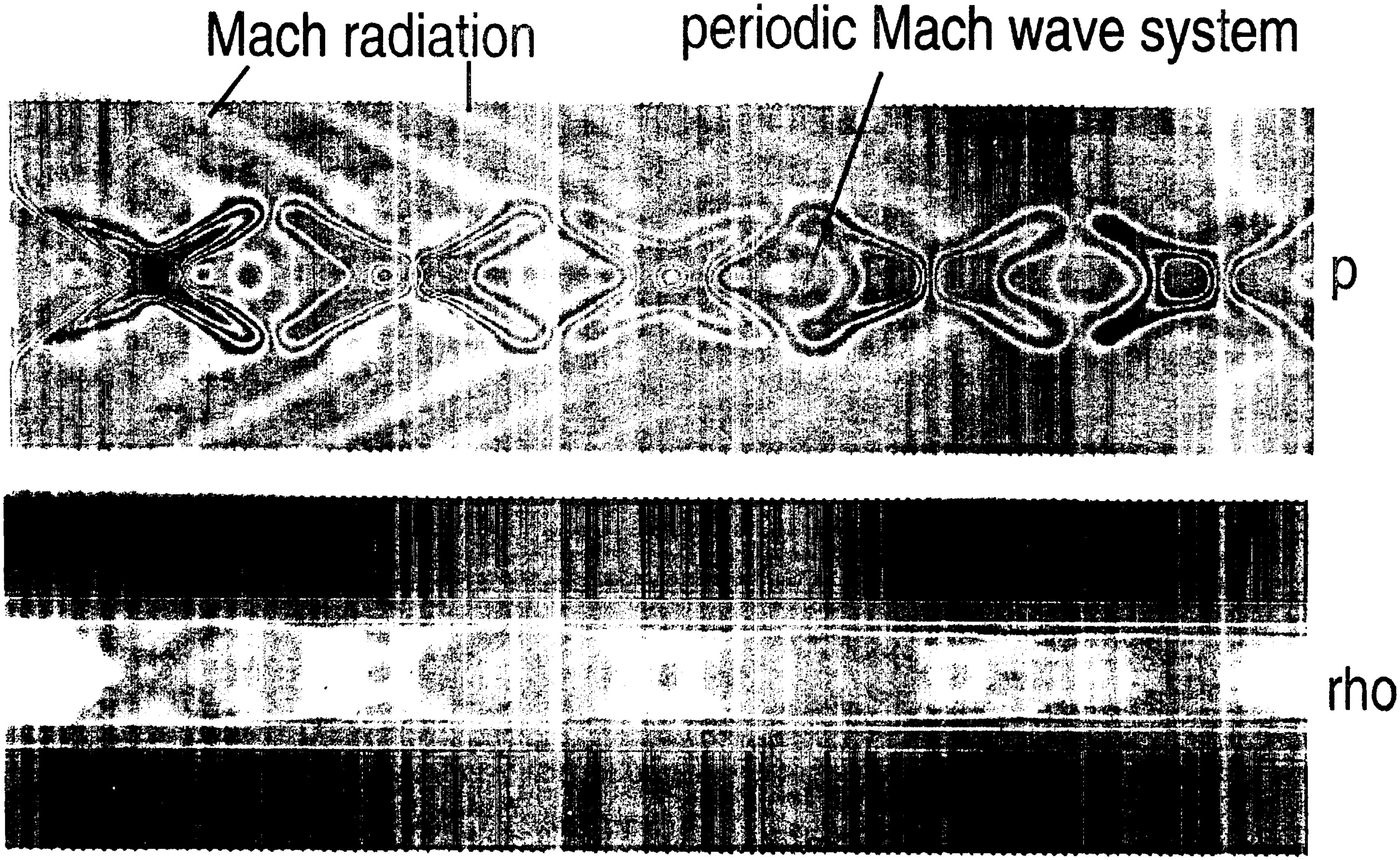
33Dx19D, $M_j = 2$, $St = 0.2$, $\epsilon = 0.001$

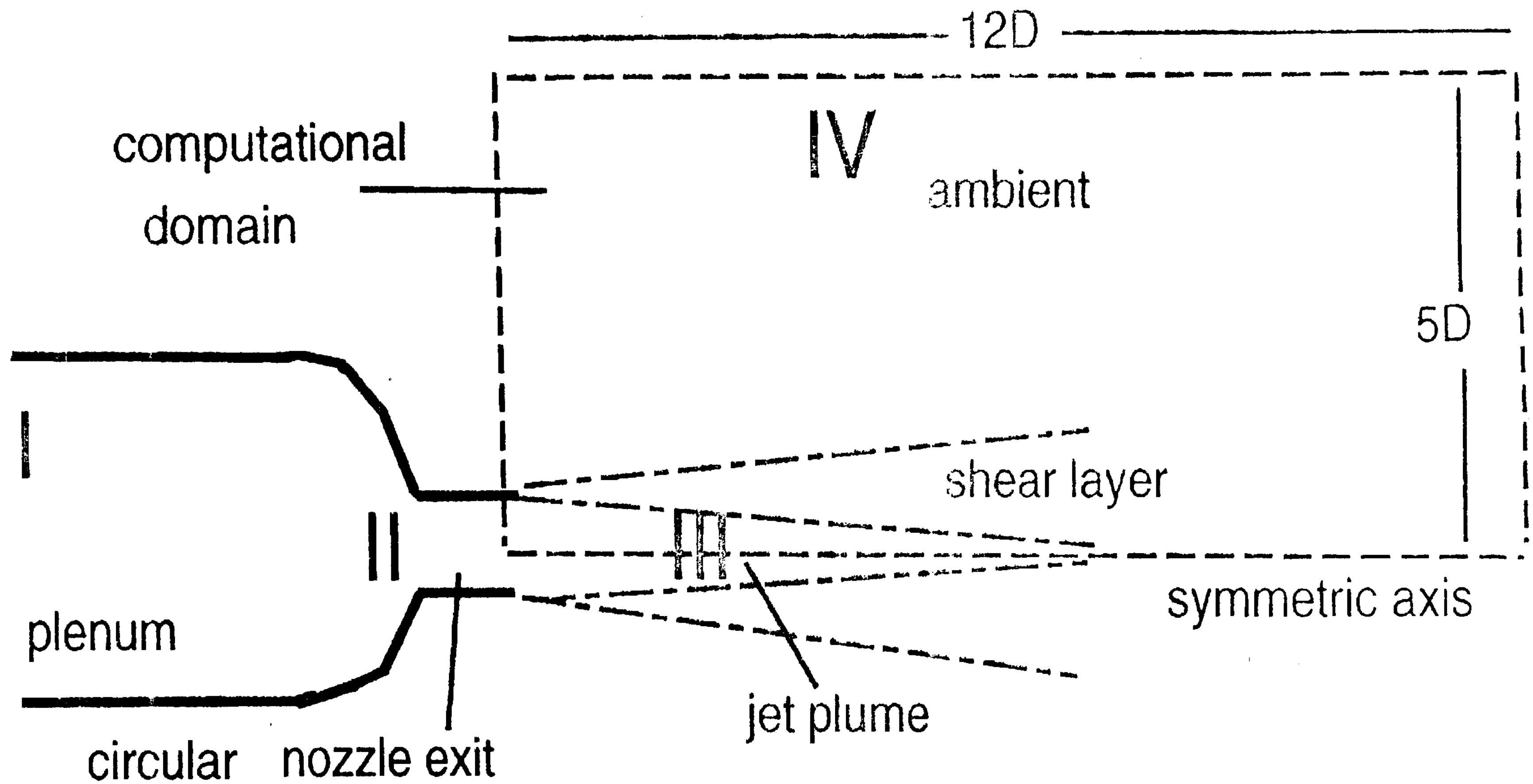
$8D \times 5D$, $M_j = 2$, $St = 0.07$, 630×150, $\delta = 10^{-4}$





M_j=1.2, 3Dx1D, 300x100 grid, St. =2, dt=.0015, ee=.00005:
showing cross-hatching Mach wave system.


Mach radiation
periodic Mach wave system
p
rho


12D
computational
domain
IV
ambient
5D
I
shear layer
II
III
symmetric axis
plenum
jet plume
circular
nozzle exit

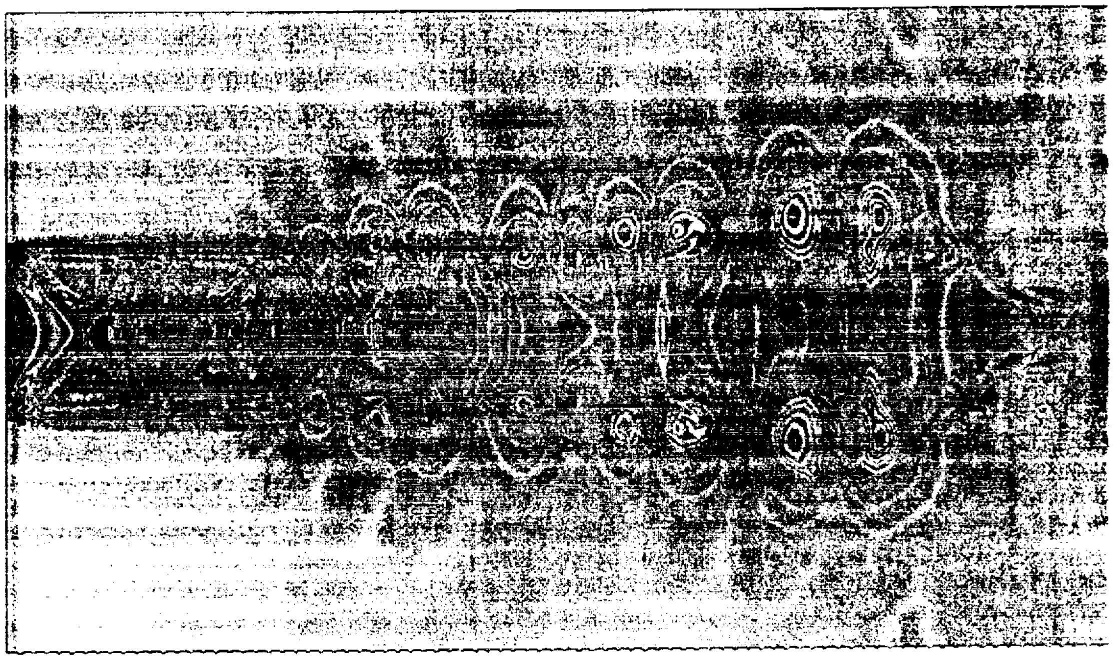


Figure 1: Numerical Schlieren by CE/SE (snapshot)

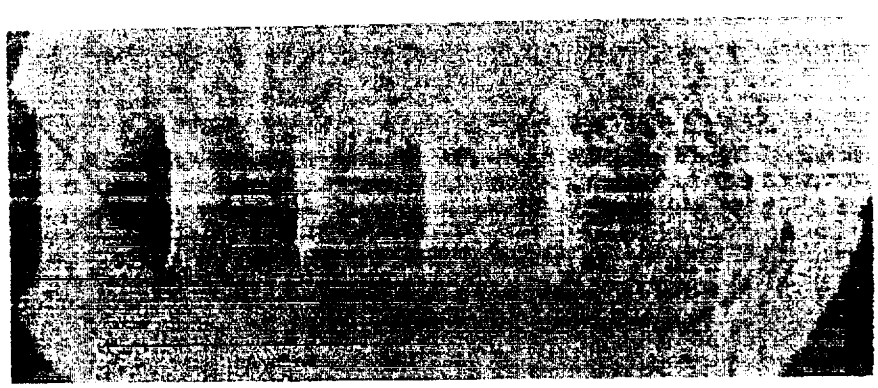
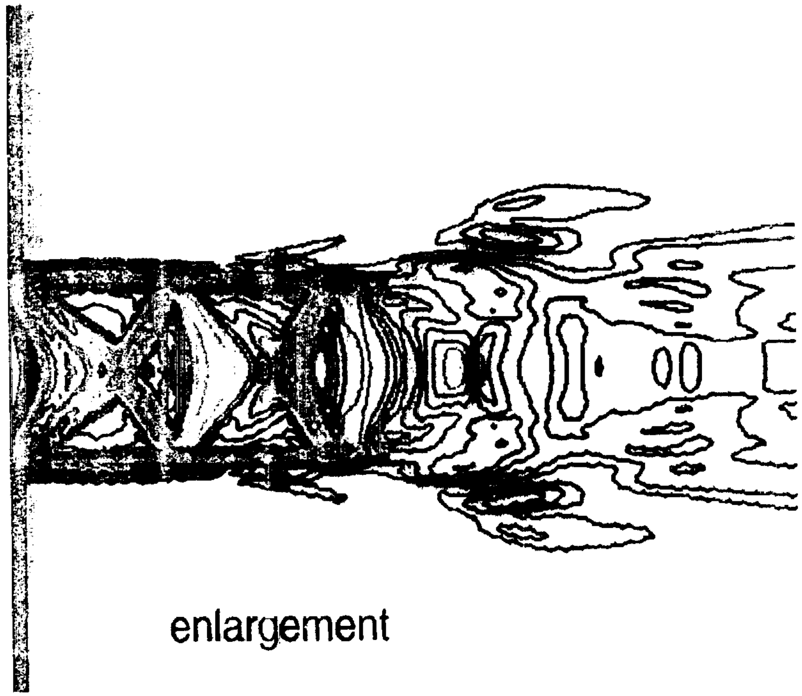
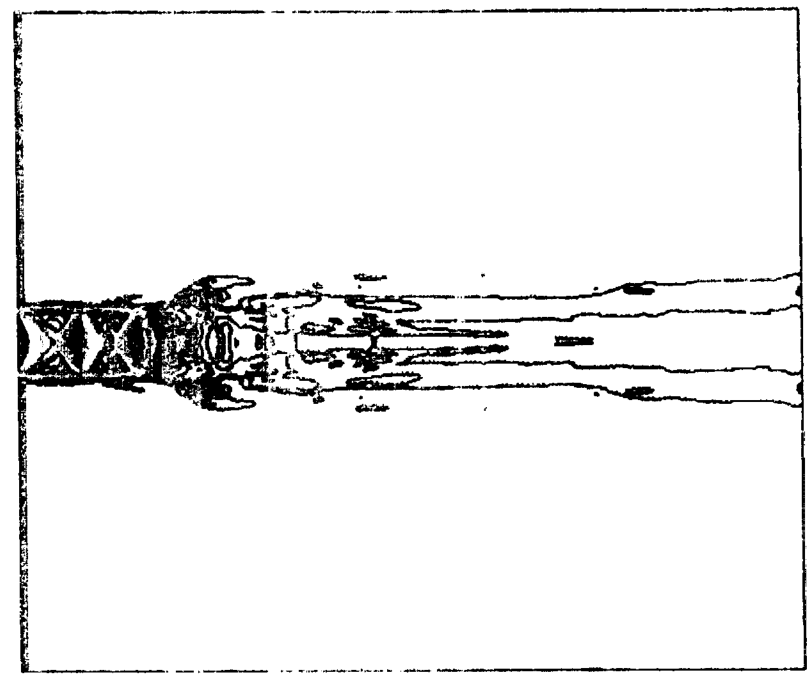
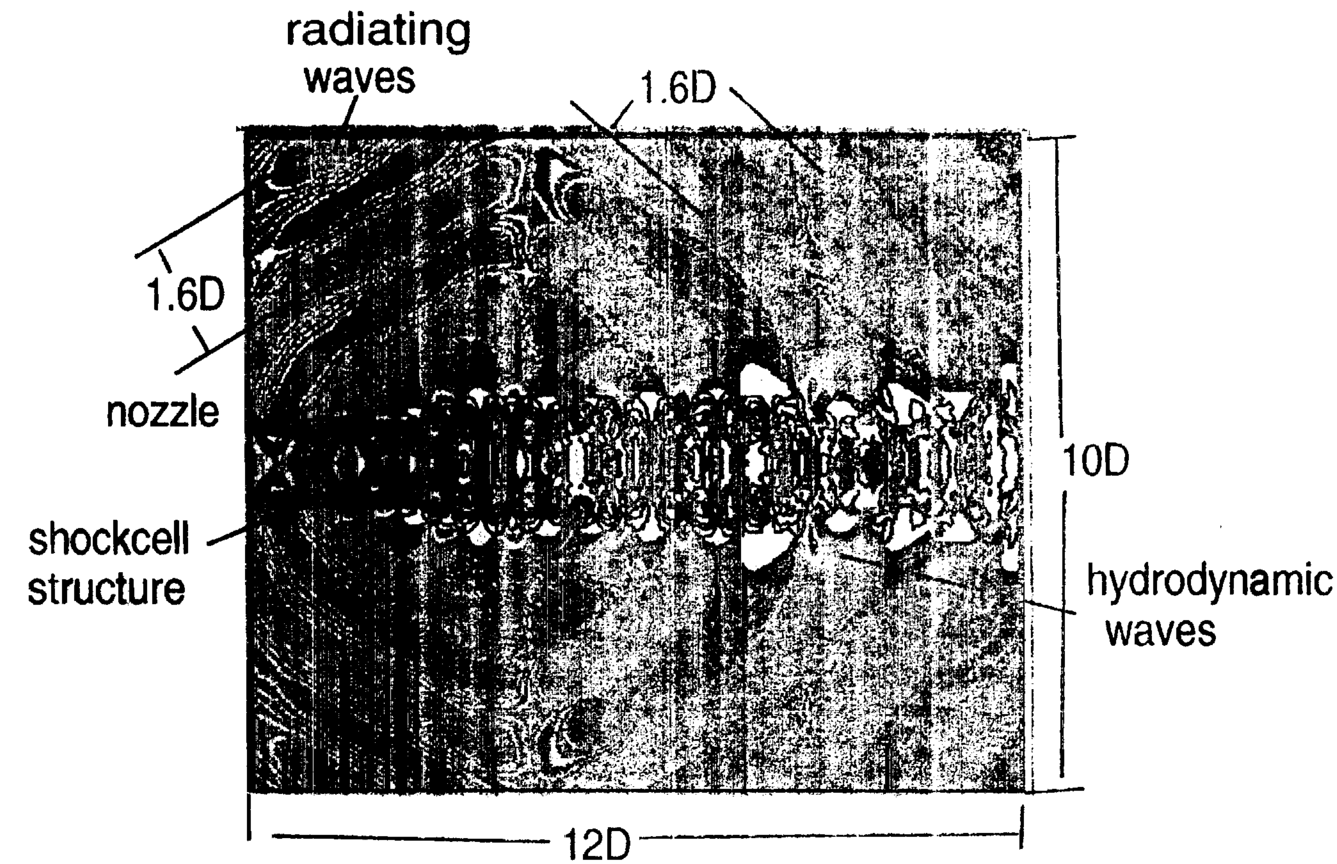


Figure 2: Experimental Schlieren (courtesy of J. Panda)

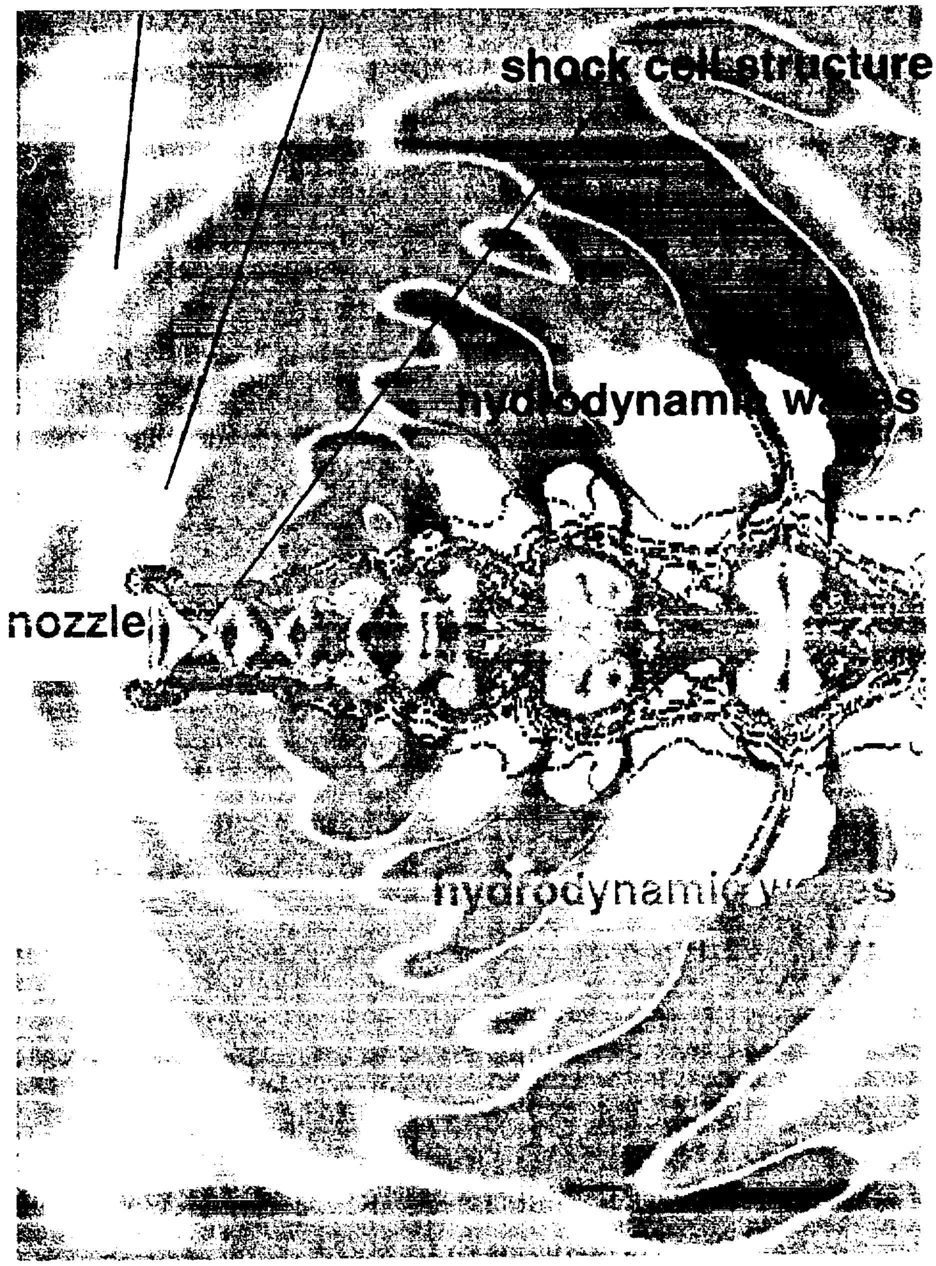


enlargement

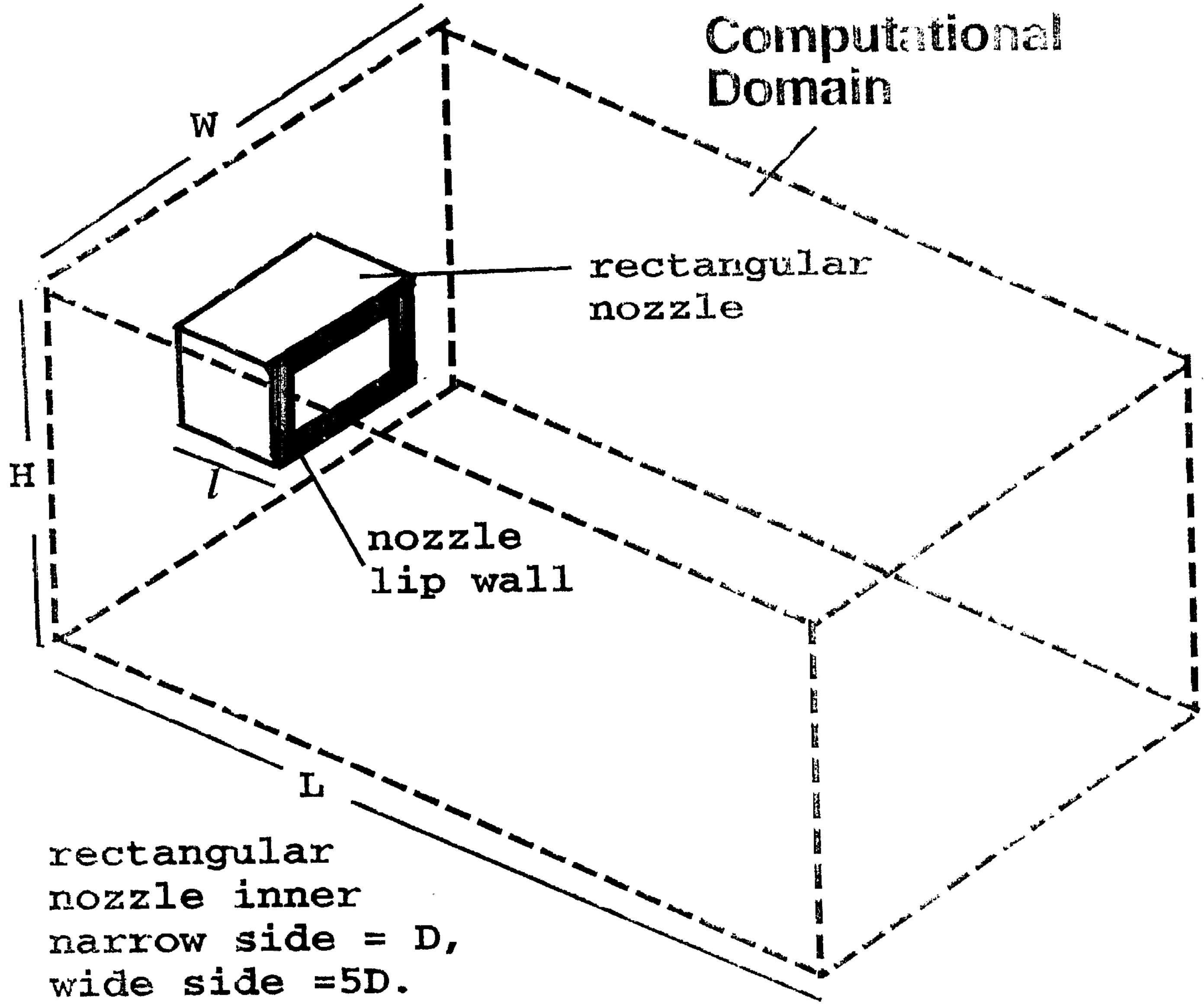


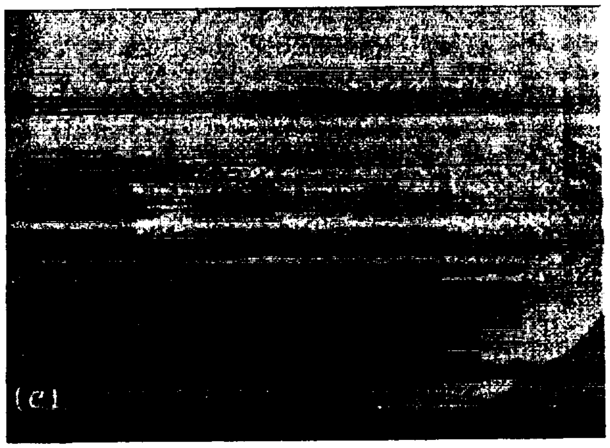


M_j=1.19 axisym. jet (underexpanded)



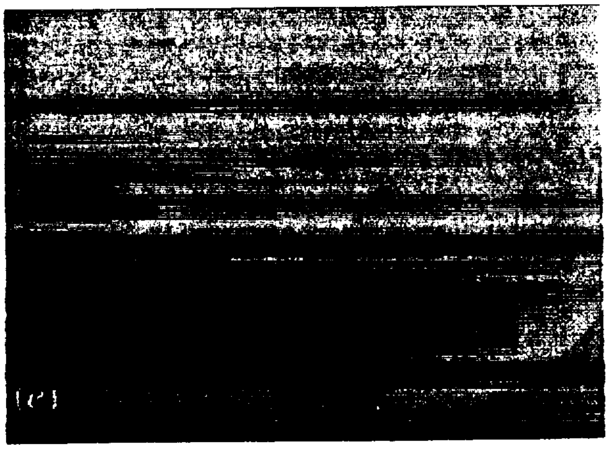
isobar and numerical Schlieren snapshots at time step=212800, underexpanded axi-sym. jet, M_j=1.19.


Computational
Domain
W
H
l
L
rectangular
nozzle
nozzle
lip wall
rectangular
nozzle inner
narrow side = D,
wide side =5D.



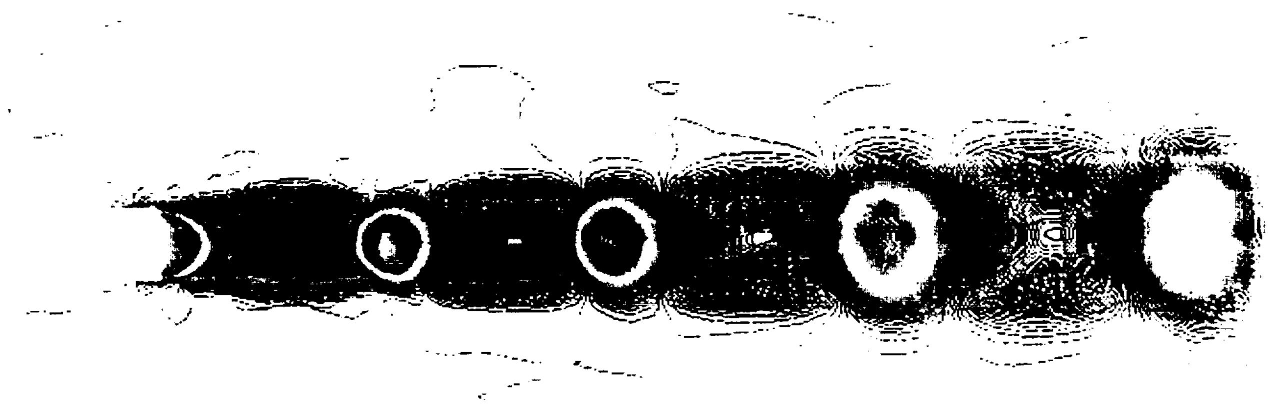
Raman's experiment

numerical Schlierens

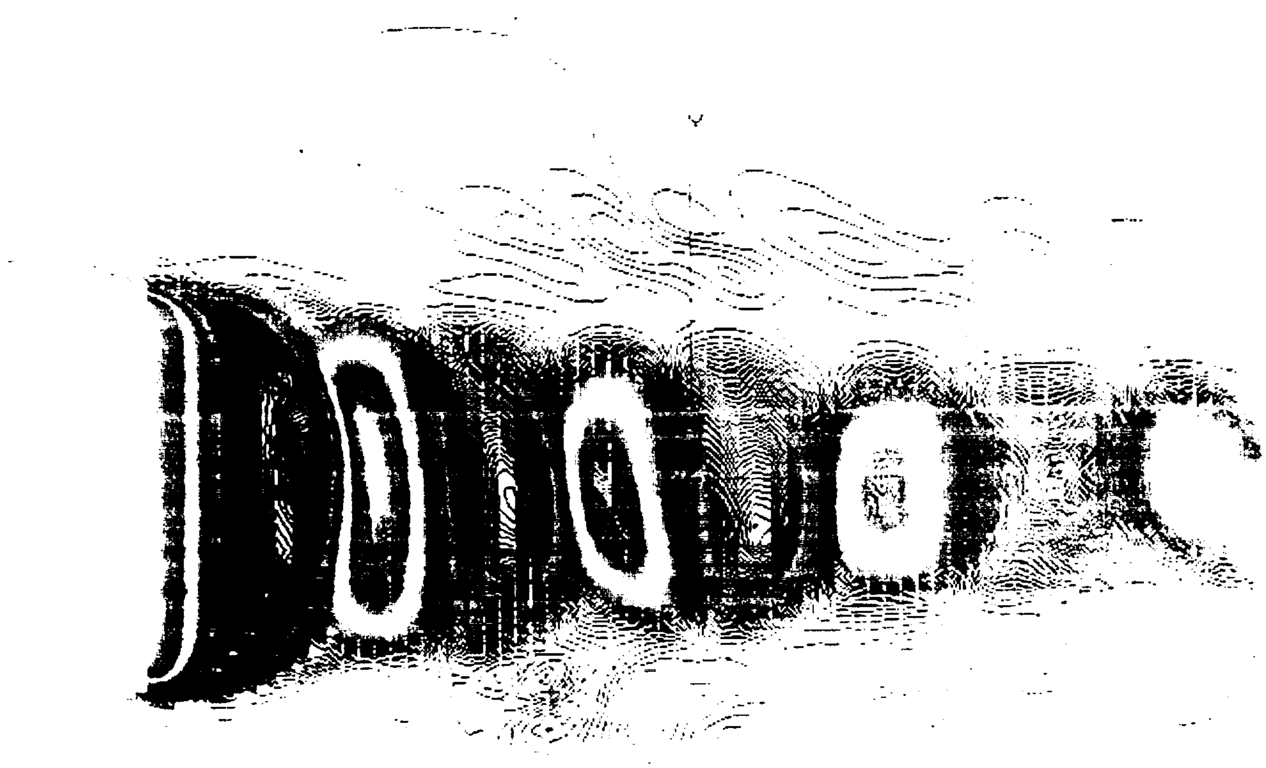


Raman's experiment

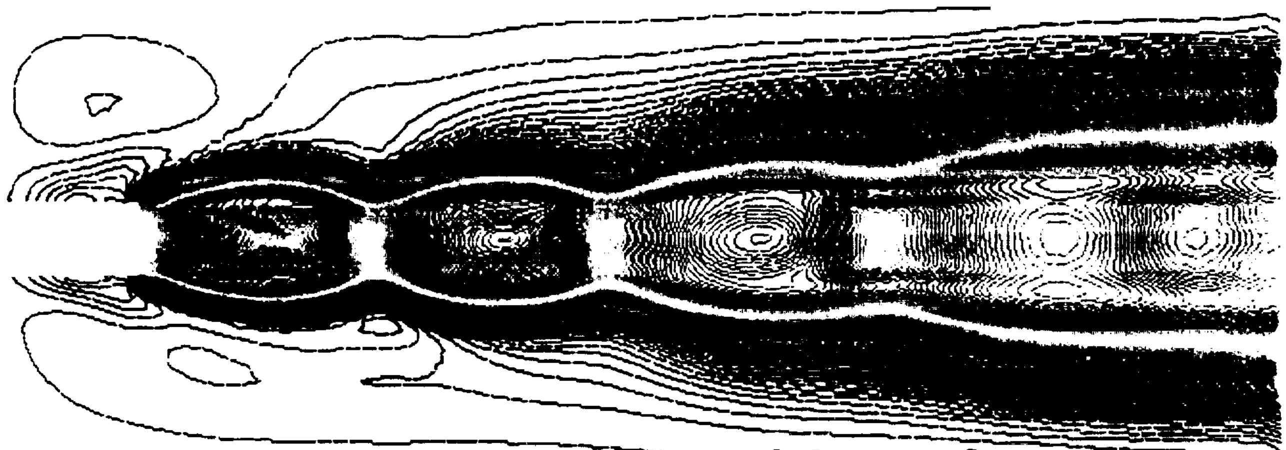
numerical Schlierens



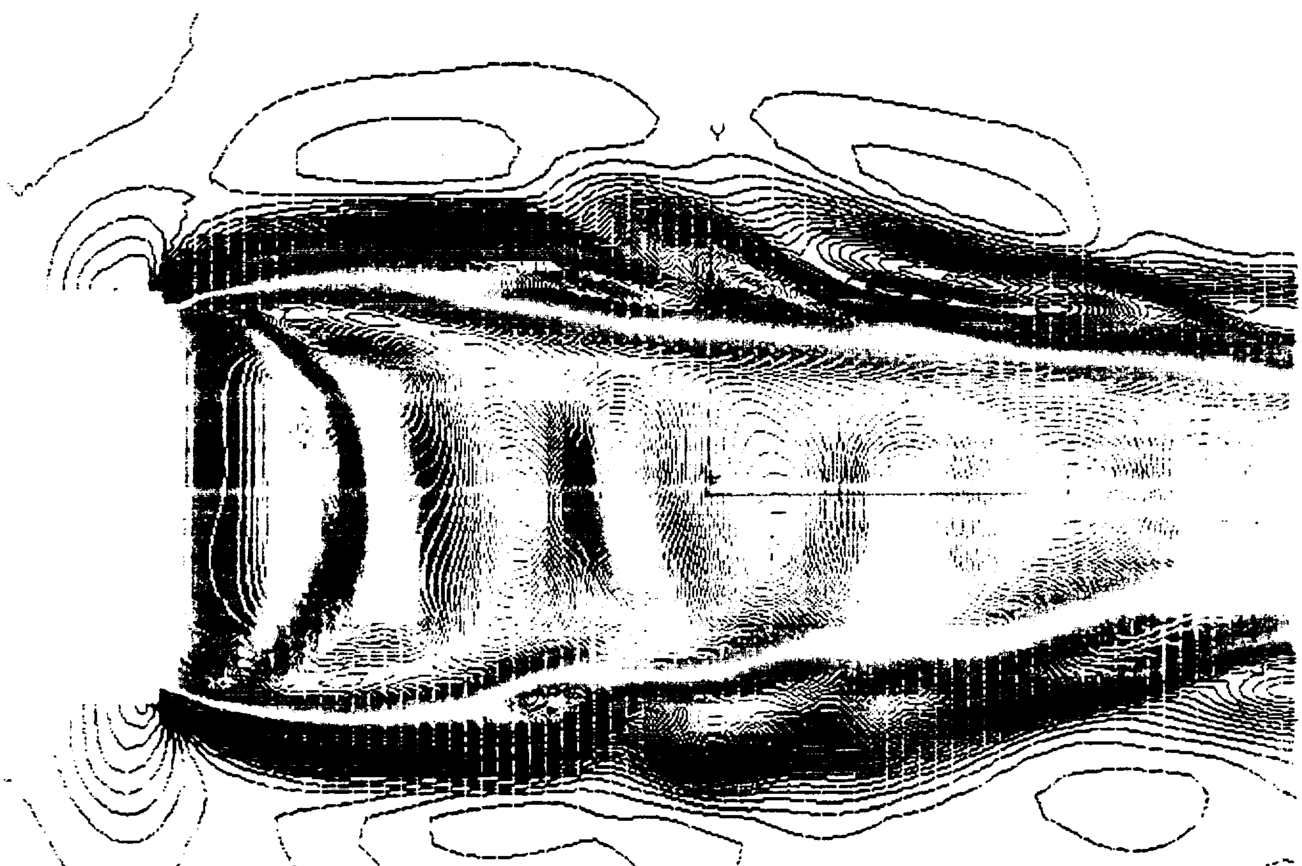
isobars, narrow side mid sect. plane



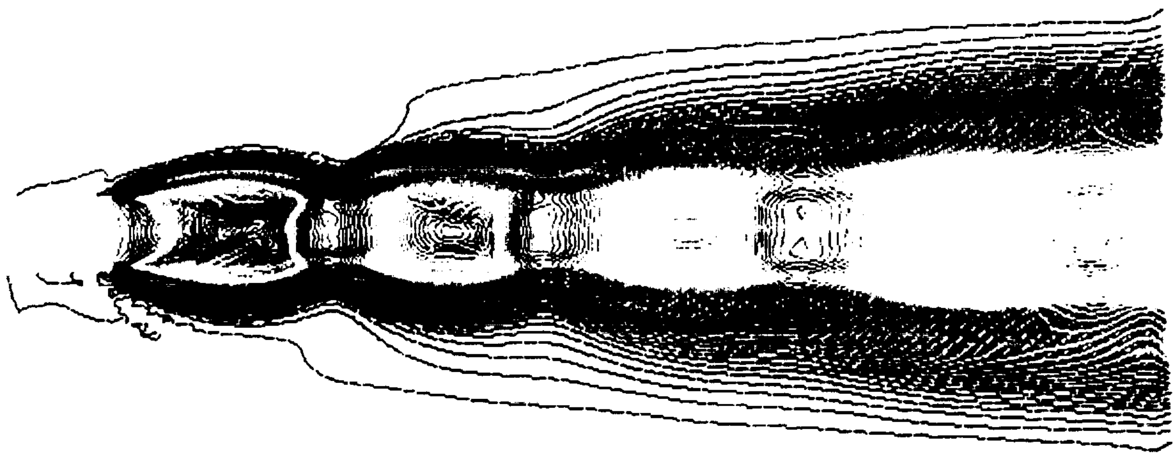
isobars, wide side mid sect. plane



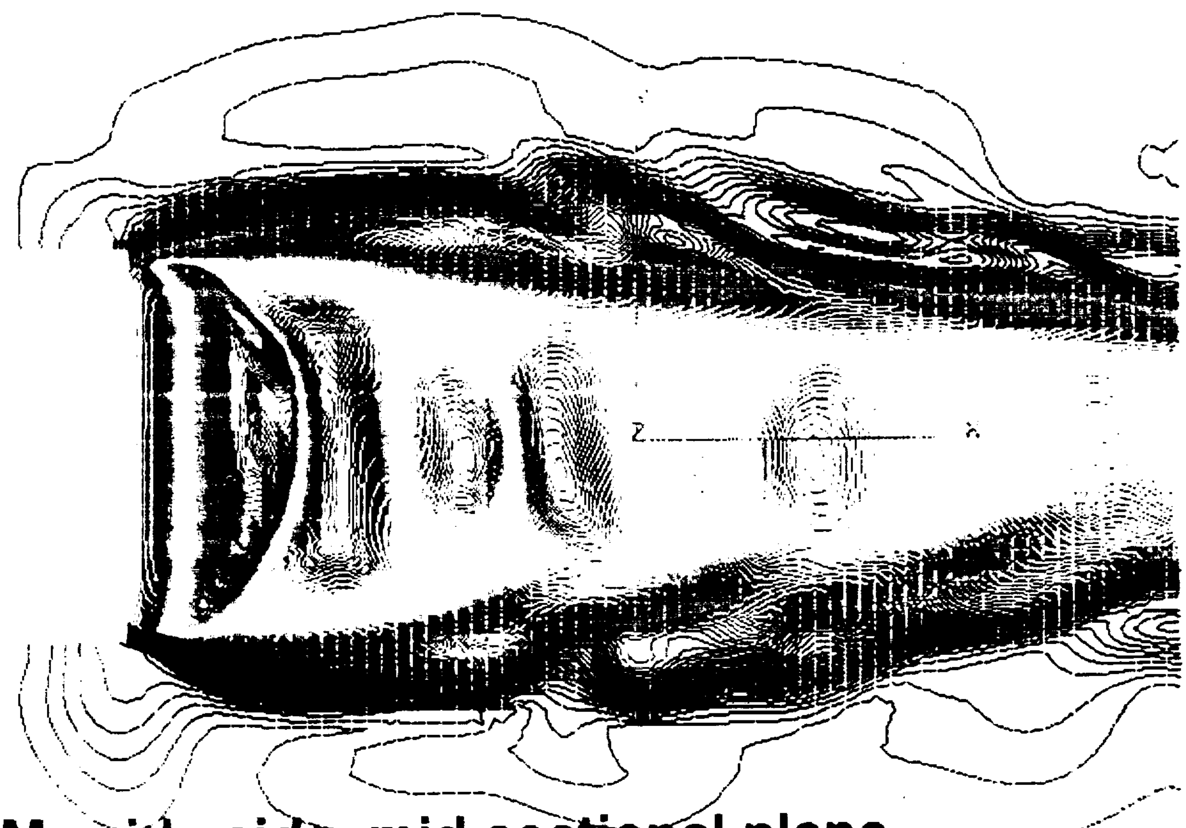
u–contours, narrow side mid sectional plane



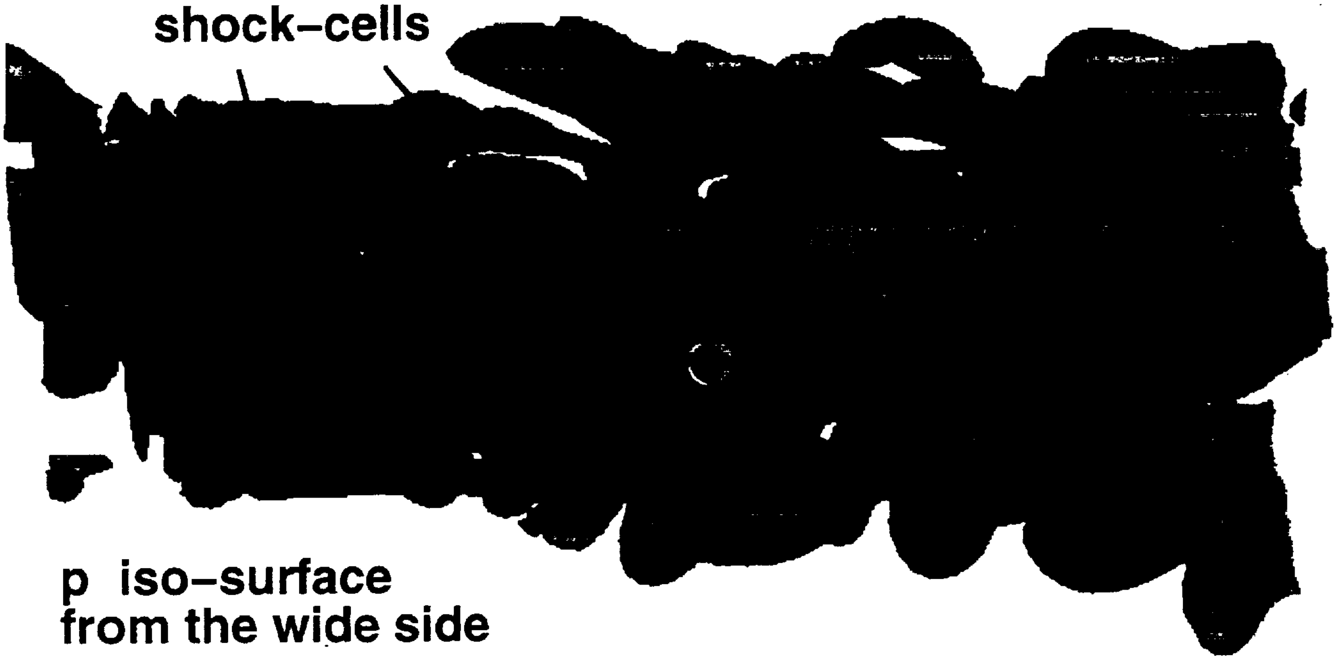
u–contours, wide side mid sectional plane



M, narrow side mid sectional plane

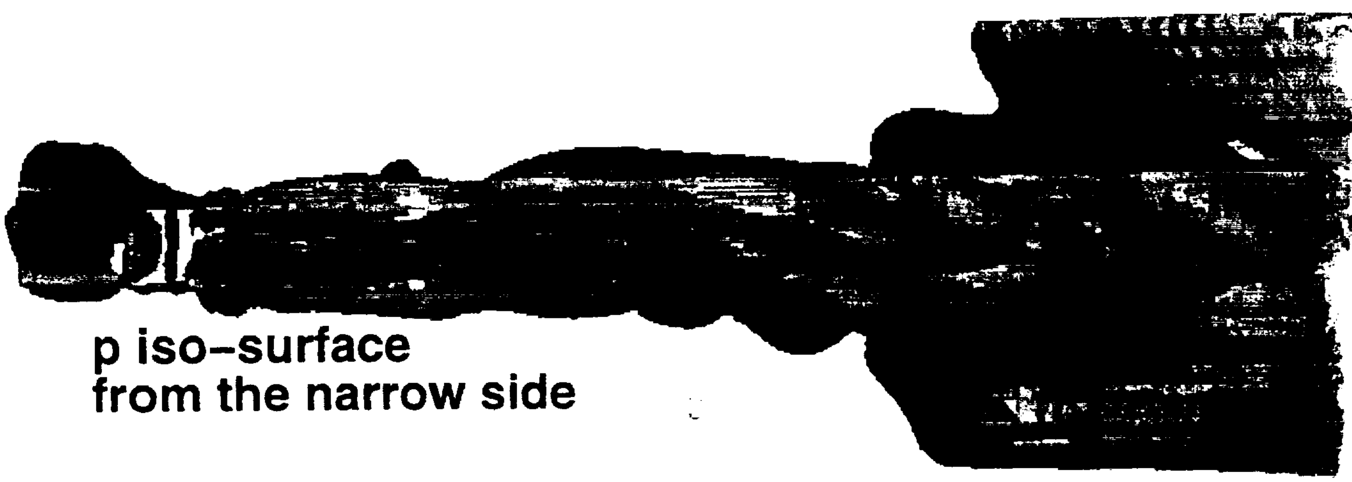


M, wide side mid sectional plane



p iso-surface
from the wide side

low level: p=.274, high level: p=.2792



p iso-surface
from the narrow side

nonlinear aeroacoustics

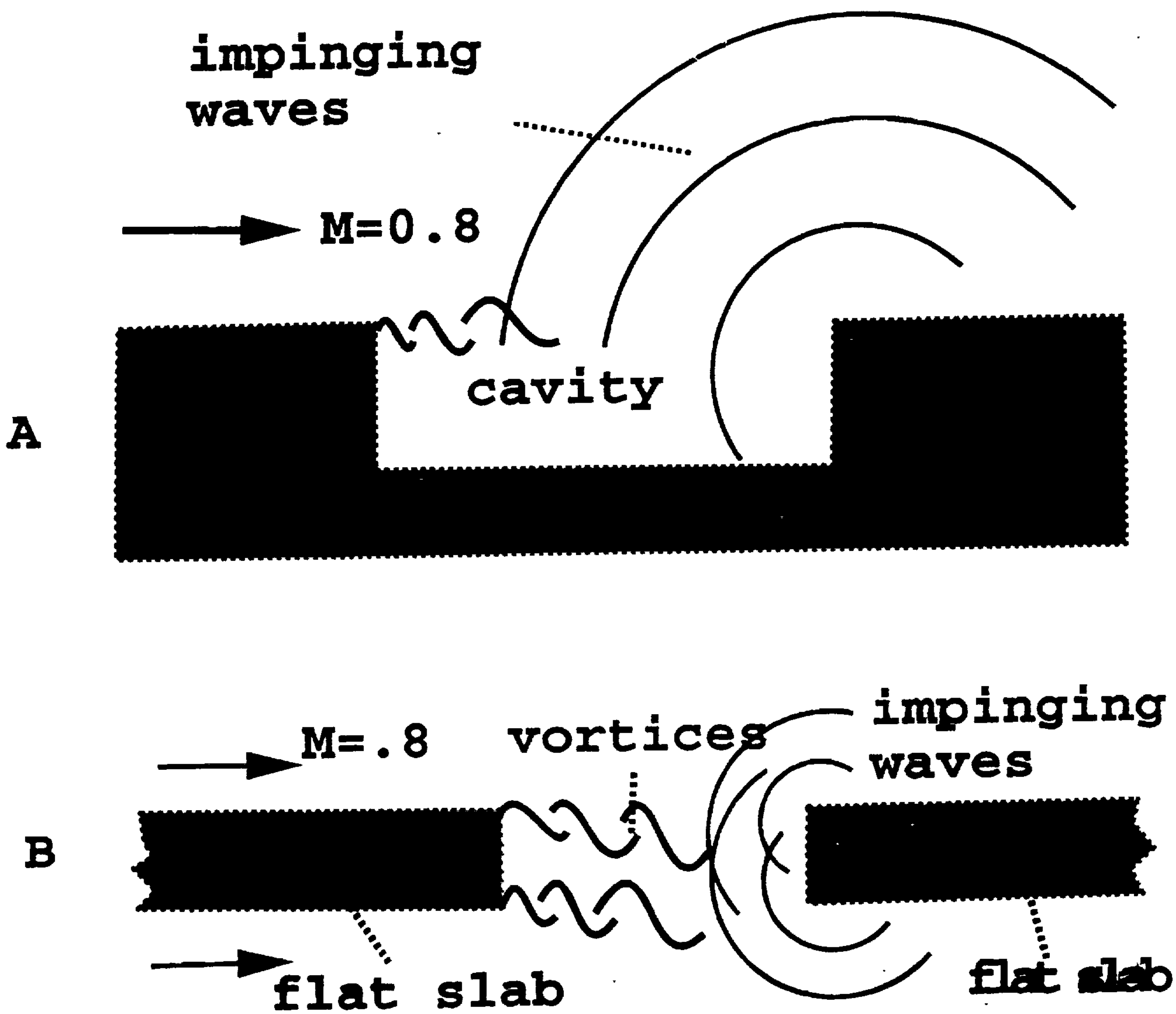
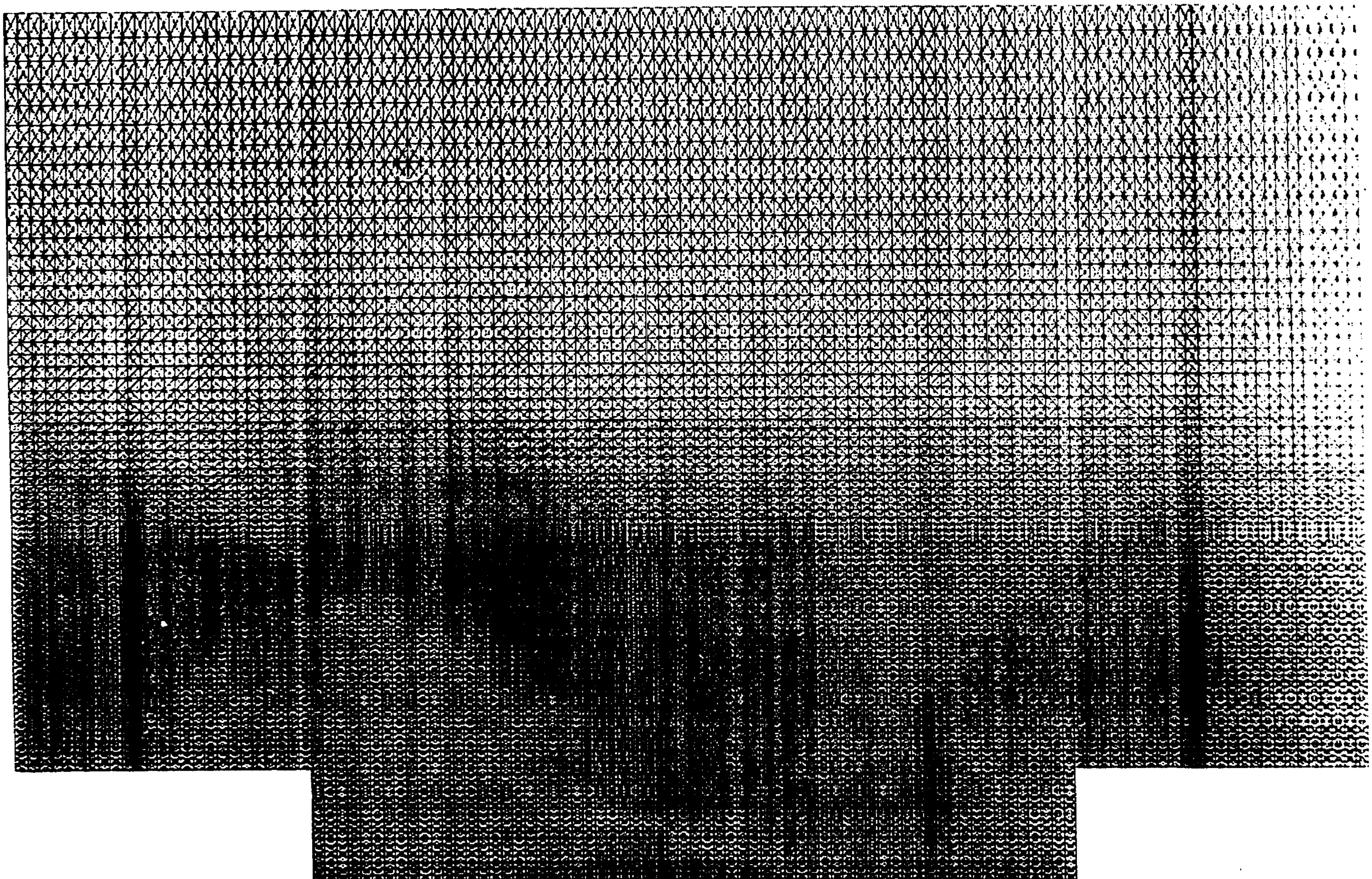
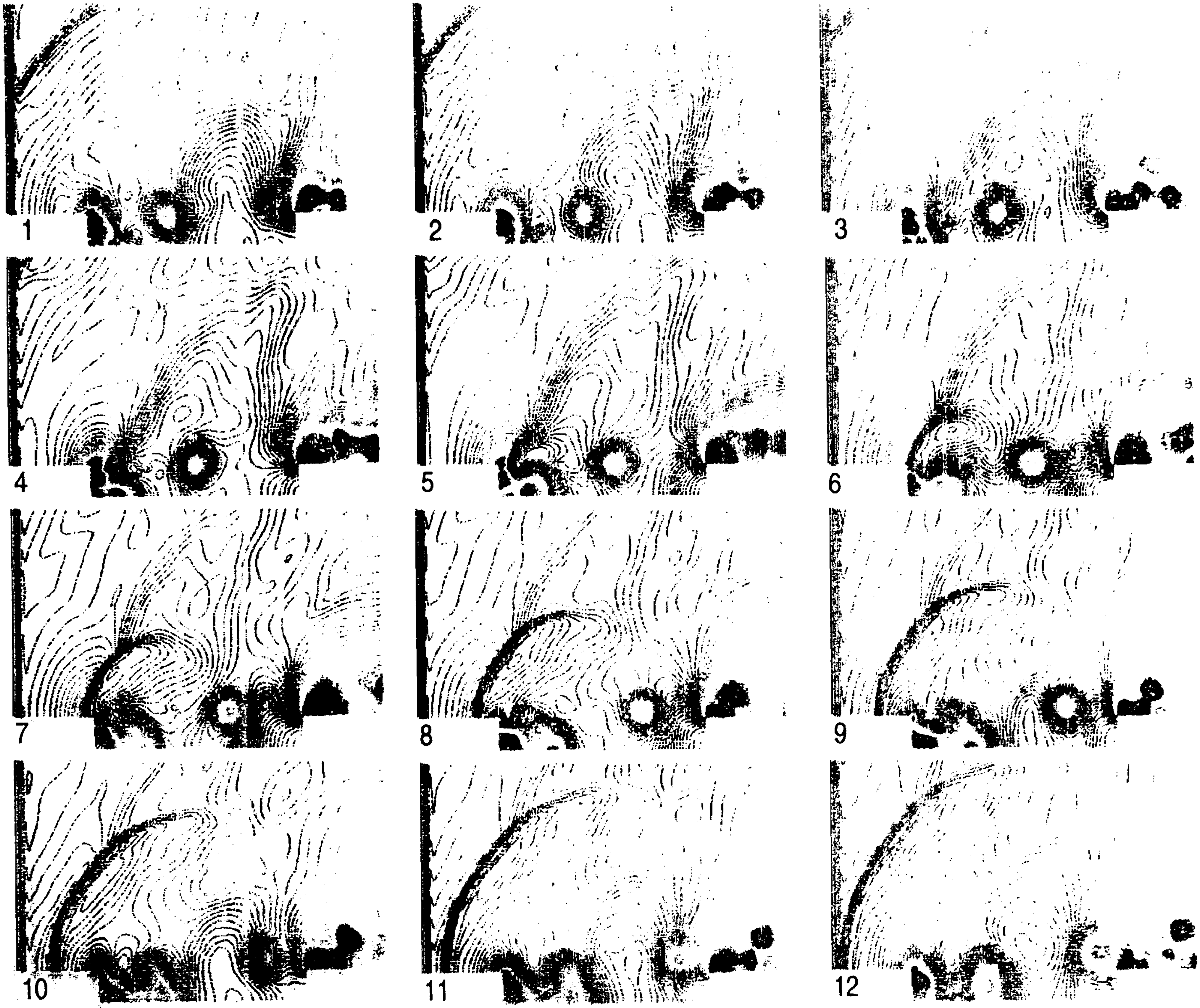


Fig. 2: Sketch of (A) cavity noise, (B) gap noise

42K elements for cavity problem





1
2
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8
9
10
11
12

Concluding Remarks

The CE/SE scheme is a time-accurate conservative scheme for CFD and CAA. It features:

- naturally adapted to unstructured grid,
- high resolution and low dispersion,
- robust, treats the 'difficult' problems in an effortless way, particularly appropriate for near field, non-linear aeroacoustics,
- the novel NRBC is simple, effective and truly multidimentional,
  (a) outflow NRBC,
  (b) transverse NRBC,
  (c) inflow absorbing NRBC.

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Your reference
Our reference
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Braunschweig, 02/11/00

Invitation for the second SWING Aeroacoustics Workshop 6-7 Oct. 2000 at DLR

Dear Dr. Loh,

In the framework of our national German CAA-project SWING we are organizing a workshop at DLR in Braunschweig in cooperation with the University of Technology Dresden. I would very much like to invite you to this second SWING workshop which will be held on 6-7 October this year. SWING (Simulation of Wing-flow Noise Generation) is an initiative geared towards the simulation of airframe noise sources at high lift devices using modern numerical CAA methods. The project partnership consists of DLR's Institute of Design Aerodynamics, the Aerodynamic Institute Aachen (Technical University of Aachen), the Institute of Aerodynamics and Gasdynamics (University Stuttgart) as well as the Institute of Acoustics and Speech Communication (University of Technology Dresden).

The aim of the workshop is to exchange knowledge on modern topics of aeroacoustics with special emphasis on numerics and to establish/deepen international connections and cooperation among research groups active in the respective field. The first SWING workshop, held in the fall of last year in Dresden, led to intensive and fruitful scientific discussions; and this is why we would like to continue this successful endeavor. During the 1.5 days of the meeting there will be half-hour presentations by the SWING project partners and by invited speakers from abroad and Germany.

It is my particular pleasure to invite you to this workshop to give a presentation about your recent research work relating to the workshop topic. Fortunately, the expenses connected with your visit and stay in Braunschweig can be covered since we are able to refund you with a bulk sum of DM 3500,- (≈ $1500,-).

We thank you for your willingness to present the space-time conservation method for aeroacoustics at the meeting as already agreed upon with Dr. Jan Delfs, who is in charge of organizing the workshop. Looking forward to seeing you in Braunschweig.

Yours sincerely,

H. Körner
H. Körner

attachment: proceedings of first SWING workshop

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